Reference Source Noise in Frequency Synthesizer

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***Abstract**: The phase noise performance of a synthesizer can be described using a simplified model. In this model we consider reference noise components that appears at the system output. The dynamic behavior of system will be analyzed by simulation using the Simulink package.*

I. Mathematical model

One measure of system performance is the steady-sate error, that is, the error remaining after transients have died out. We will be calculated and represented the transient phase error sample sequence as a function of the system parameters.

A synthesizer block diagram for analyzes the output error we purpose in figure 1.

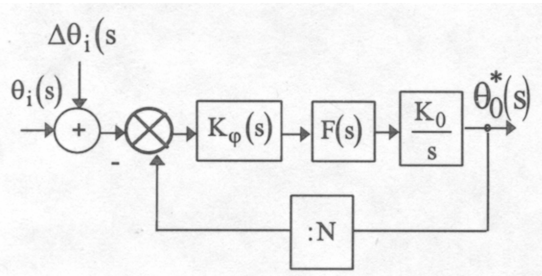

Figure 1.

where:

$K_\varphi(s)$ - is the phase detector gain factor,

$F(s)$ - the filter transfer function,

K_0 - the controlled oscillator gain factor,

$\frac{1}{N}$ - the programmable divider transfer function,

$\Delta\theta_0(s)$ - the reference noise and,

θ_0^* - is the output signal phase in the presence of the reference noise.

We can write:

$$\theta_0^* = \frac{K_0 K_\varphi(s) F(s)}{s}\left[\theta_i(s) + \Delta\theta_i(s) - \frac{\theta_0^*}{N}\right] \quad (1)$$

or:

$$\theta_0^* = \frac{K_0 K_\varphi(s) F(s)}{s + \frac{K_0 K_\varphi(s) F(s)}{N}}\left[\theta_i(s) + \Delta\theta_i(s)\right] \quad (2)$$

Let us assume that:

$$\theta_0^* = \theta_0(s) + \Delta\theta_{0i}(s) \quad (3)$$

In equation (3) $\Delta\theta_{0i}(s)$ is the portion reference noise, which appears at the system output.

The transfer function from the reference phase noise disturbance to the out of loop is given by:

$$\frac{\Delta\theta_{0i}(s)}{\Delta\theta_i(s)} = \frac{K_0 K_\varphi(s) F(s)}{s + \frac{K_0 K_\varphi(s) F(s)}{N}} \quad (4)$$

If the phase detector is a three-states phase-frequency detector, we have:

$$K_\varphi(s) = K_\varphi \quad (5)$$

The typical loop filter uses the form:

$$F(s) = \frac{1 + s\tau_1}{s\tau_2} \quad (6)$$

In this case, expression (4) becomes:

$$\frac{\Delta\theta_{0i}(s)}{\Delta\theta_i(s)} = N\frac{2\xi\omega_n s + \omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} \quad (7)$$

where:

$$\omega_n = \sqrt{\frac{K_0 K_\varphi}{N\tau_2}} \tag{8}$$

is the natural frequency for the control loop, and:

$$\xi = \frac{1}{2}\omega_n \tau_1 \tag{9}$$

is the damping factor.

Classical Laplace transform techniques may be used for continuous system. Sampled-data phase locked loops have played a significant role in frequency synthesis ever since digital elements (dividers, phase detectors) began supplementing the analog design process. In this context, we can use a technique for converting a rational Laplace transform into a representative z transform. The method is based on approximating s^{-1} as:

$$s^{-1} = \frac{T}{2}\frac{z+1}{z-1} \tag{10}$$

It is interesting to determine the output phase error $\Delta\theta_{0i}(z)$.

We obtain the equation for error:

$$\frac{\Delta\theta_{0i}(z)}{\Delta\theta_i(z)} = \frac{K_1 z^2 + K_2 z + K_3}{K_0 z^2 + M_1 z + M_2} \tag{11}$$

where:

$$\begin{cases} K_0 = 1 + \xi\omega_n T + \left(\omega_n \frac{T}{2}\right)^2 \\ K_1 = \left[\xi\omega_n T + \left(\omega_n \frac{T}{2}\right)^2\right]\cdot N \\ K_2 = \frac{(\omega_n T)^2}{2}\cdot N \\ K_3 = \left[-\xi\omega_n T + \left(\omega_n \frac{T}{2}\right)^2\right]\cdot N \end{cases} \tag{12}$$

and:

$$\begin{cases} M_1 = -2 + \frac{(\omega_n T)^2}{2} \\ M_2 = 1 - \xi\omega_n T + \left(\omega_n \frac{T}{2}\right)^2 \end{cases} \tag{13}$$

Recognizing z^{-1} as the simple unit delay operator may rewritten in the time domain as the recursion:

$$\Delta\theta_{0i}(n) = \frac{1}{K_0}\left[K_1\Delta\theta_i(n) + K_2\Delta\theta_i(n-1) + K_3\Delta\theta_i(n-2)\right] - \frac{1}{K_0}\left[M_1\Delta\theta_{0i}(n-1) + M_2\Delta\theta_{0i}(n-2)\right] \tag{14}$$

The dynamic behavior of the system can be analyzed by using the Simulink package. In figure 2 is shown the model for our case.

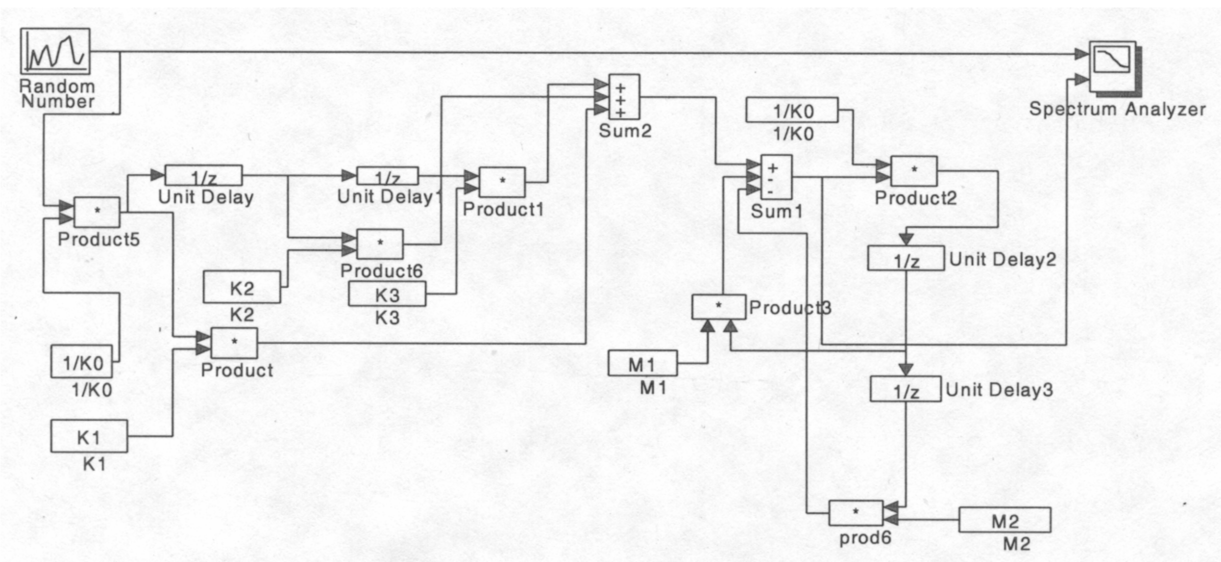

Figure 2.

II. Numerical results

Equation (14) may be used to calculate the phase error response. Figure 3 describes the spectrum of input signal.

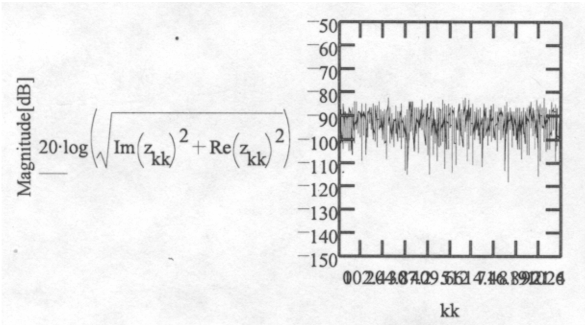

Figure 3.

The results of the output spectrum are shown graphicaly in cases: figure 4 for N=1000; $\xi = 0.7$; $\omega_n T = 0.01$ and figure 5 for N = 1000; $\xi = 0.7$; $\omega_n T = 0.001$.

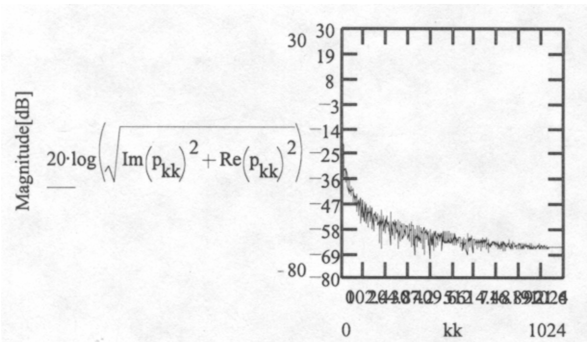

Figure 4.

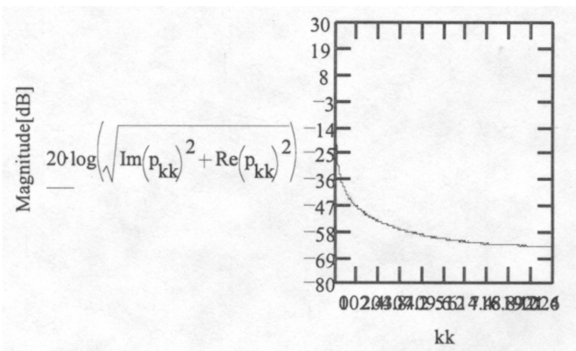

Figure 5.

Results of simulating system's behavior for the case N = 1000; $\xi = 0.7$; $\omega_n T = 0.001$ are presented in figure 6.

The system behaves as a low-pass filter with respect the phase noise of the reference signal. The components of reference noise with rates below natural frequency ω_n appear

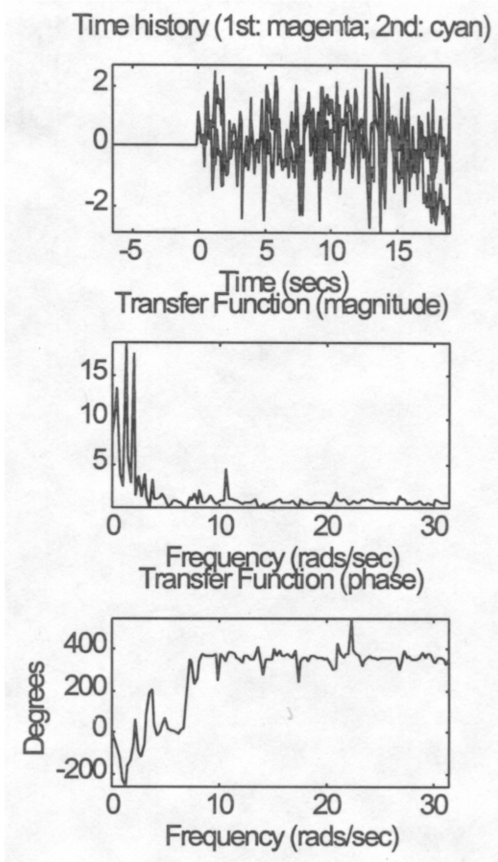

Figure 6.

at the output whereas components with rates above ω_n are attenuated by the loop.

We can conclude that the output noise power is the reference noise power multiplied by N2.

III. References

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