Simulation of the Local Model Reference Adaptive Control of the Robotic Arm with D.C. Motor Drive

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Abstract: *Adaptive controllers are used for plants with unknown or time varying parameters. The robotic arm is a classical case of plant with unknown parameters because the load modifies in time. Using the Lyapunov theory for the parameters adaptation law assures the global stability and robustness of the system and superior performances in presence of the disturbance or in case of plant parameters changes. An alternative to global control of the robot is the local compensation of the load and interdependencies for every arm, which means a D.C. motor adaptive control problem.*
The paper presents the design of the local adaptive control of the robotic arm driven by a D.C. motor using Lyapunov stability theory. The globally model of the designed system is implemented in Matlab Simulink and simulated to different references.

Keywords: *model reference adaptive control, adaptive controllers, D.C. motor, robot control.*

1. INTRODUCTION

For process with unknown or time varying parameters and known structure, in presence of great disturbances a robust solution is offered by the adaptive systems. Depending on the type of the real plant a model reference adaptive system or a self-tuning controller, continuous or discrete, is chosen. Excepting the intelligent systems, the adaptive systems are the only ways to compensate the large variations of the plant parameters. [1,2]

The actual tendency in study of automatic systems is to replace the classic controllers with intelligent and adaptive systems, especially where the desired performances and the technical demands are not satisfied.

The first research in adaptive systems domain was made around 1950. At that time the studies were focused on automatic pilot for high performance airplanes. In time Whitaker, Bellman, Landau, Kalman, Astrom, Wittenmark and others contributed to adaptive system theory progress and industrial implementation. [3,4]

The serial robots have a kinematic chain with a serial construction, which involves many influences between the forces and moments of the various degrees of freedom. So, the dynamic model of the robot has nonlinearities and strong variations of process parameters. The main cause generating these design issues of regulators is the interdependence between different axes of movement, a serial robot typical phenomenon.

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If we assume that we are dealing with a robot driven by D.C. servomotors, then we can achieve an axes decoupled position control, dedicated to each arm of the serial robot. Robotic arm parameters variations are compensated locally in this case by the model reference adaptive control.

2. THEORETICAL ASPECTS ON MODEL REFERENCE ADAPTIVE CONTROL

When the plant parameters and the disturbance are varying slowly, or slower than the dynamic behavior of the plant, then a MRAC control scheme can be used. This adaptive structure offers a superior performance and robustness in time than a classical PID controller.

The model reference adaptive control scheme is shown in figure 1. [1,3]

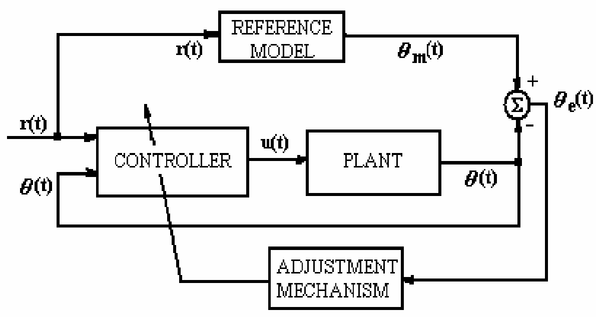

Fig. 1. Model reference adaptive control scheme.

The MRAC structure consists of four main parts: the plant, the controller, the reference model and the adjustment mechanism. [5,8].

In MRAC, the technical demands and the desired input-output behavior of the closed loop system is given via the corresponding dynamics of the reference model. Therefore, the basic task is to design such a control, which will ensure the minimal error between the reference model and the plant output (adaptation error) despite the uncertainties or variations in the plant parameters and working conditions.

The adjustment mechanism uses this adaptation error θ_e to adjust the controller parameters. The adaptation (tracking) error $\theta_e=\theta_m-\theta$ represents the deviation of the plant output from the desired trajectory. The adaptation laws for the controller parameters are determined using Lyapunov theory of stability. The Lyapunov theory assures that the adaptation error zero is asymptotically stable (robustness of the system).

3. MODELLING THE LOCAL ADAPTIVE CONTROL OF THE GEAR DRIVEN ROBOT ARM

We suppose that each flexible-link of the robot is gear driven by a D.C. servomotor. So, our plant used in simulation is in fact a D.C. servomotor. The dynamic equations of the D.C. servomotor are:

$$\begin{cases} u(t) = R\cdot i(t) + L\cdot di(t)/dt + k_e\cdot\Omega_{dc}(t) \\ k_m\cdot i(t) = J\cdot d\Omega_{dc}(t)/dt + B\cdot\Omega_{dc}(t) + C_r(t) \\ \Omega_{dc}(t) = d\theta_{dc}(t)/dt, \theta(t) = \mu\cdot\theta_{dc}(t) \end{cases} \quad (1)$$

Where the plant parameters are voltage u, current i, circuit resistance R, circuit inductance L, electromotive voltage $k_e\Omega$, motor torque $k_m i$, inertia J, viscous frictional coefficient B, load torque C_r, rotational speed Ω_{dc}, reduction ratio μ, motor angle θ_{dc}, gear angle θ. [9]

The plant is described by a third order differential equation and has as input the voltage u and as output the rotational gear angle θ. Because the electrical time constant of the D.C. motor is very small relative to his electromechanical time constant and to simplify the adaptive controller design the plant model will be approximate with a second order system. The load torque can be considered a disturbance.

Considering $C_r=0$ and L very small the input-output simplified model of the D.C. motor is given by relation (2).

$$H(s) = \frac{\theta(s)}{U(s)} = \frac{k_m\cdot\mu}{s\cdot[k_e k_m + R\cdot(B+s\cdot J)]} \quad (2)$$

Figure 2 shows the simplified simulink model for one robotic arm gear driven with D.C. motor and the step unit response. The step response highlights the good approximation of the third order model with the second order system. [7]

The reference model for the MRAC generates the desired trajectory, θ_m, which the plant output θ has to follow. A standard second order differential equation was chosen as the reference model, with the following coefficients: $a_m=1, b_m=1$.

$$d^2\theta_m/dt^2 + a_m\cdot d\theta_m/dt + b_m\cdot\theta_m = b_m\cdot r \quad (3)$$

and using Laplace transform the continuous transfer function of the reference model is given by relation (4).

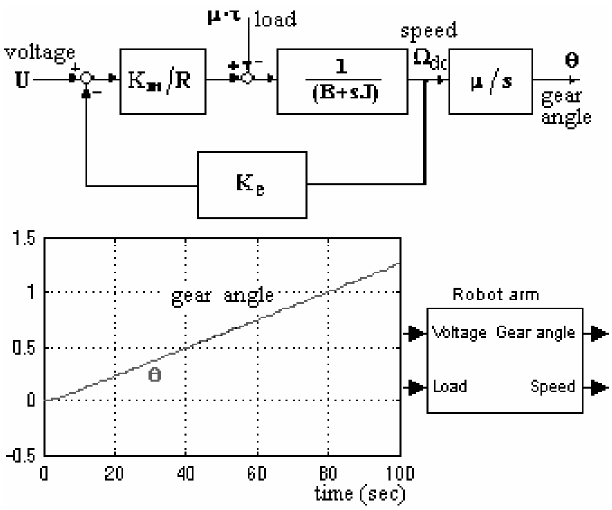

Fig. 2. Simulink simplified model of a robotic arm.

$$H_m(s) = \frac{\theta_m(s)}{R(s)} = \frac{b_m}{s^2 + a_m\cdot s + b_m} \quad (4)$$

Due to the specifics of the plant and the reference model (second order model), the feedforward-feedback controller for the D.C. motor gear driven has a fixed structure with three parameters k_1, k_2, k_3. The control law for the MRAC structure takes the following form:[9]

$$u(t) = k_1\cdot r(t) - k_2\cdot\theta(t) - k_3\cdot\dot{\theta}(t) \quad (5)$$

For $k_1=k_2$ the controller becomes a proportional derivative (PD) controller with the derivative component on the feedback loop. [9]

The Lyapunov stability theory which has the final task to determinate the adaptation laws for the controller parameters need the input-state-output models of the closed-loop and model reference systems. The state equations for these systems result from the relations (1), (2) and (3).

$$\begin{cases} \dot{x}(t) = A(t)\cdot x(t) + B(t)\cdot r(t) \\ \dot{x}_m(t) = A_m\cdot x_m(t) + B_m\cdot r(t) \end{cases} \quad (6)$$

where the new variables and the systems matrix are:

$$x = \begin{bmatrix} \theta & \dot{\theta} \end{bmatrix}^T, x_m = \begin{bmatrix} \theta_m & \dot{\theta}_m \end{bmatrix}^T$$

$$A(t) = \begin{bmatrix} 0 & 1 \\ -\frac{k_m\cdot\mu\cdot k_2(t)}{R\cdot J} & -\frac{k_m\cdot k_m + k_m\cdot\mu\cdot k_3(t)}{R\cdot J} \end{bmatrix}$$

$$B(t) = \begin{bmatrix} 0 \\ \frac{k_m\cdot\mu\cdot k_1(t)}{R\cdot J} \end{bmatrix}$$

$$A_m = \begin{bmatrix} 0 & 1 \\ -b_m & -a_m \end{bmatrix}, B_m = \begin{bmatrix} 0 \\ b_m \end{bmatrix}$$

The dynamics of the adaptation error is described by a second order system with the state equation (7).

$$\dot{x}_e(t) = A_m \cdot x_e(t) + \left(A_m - A(t)\right)\cdot \theta(t) + \\ + \left(B_m - B(t)\right)\cdot r(t) \tag{7}$$

where x_e is a 2x1 vector, containing the adaptation error and its derivative $x_e = \begin{bmatrix} \theta_e & \dot{\theta}_e \end{bmatrix}^T$.

In the Lyapunov stability theory the first major problem is to choose the proper positive definite function $V(t,x_e)$. [4,5,6]

$$V = x_e^T P x_e + tr\left\{(A_m - A)^T \gamma_A (A_m - A)\right\} + \\ + tr\left\{(B_m - B)^T \gamma_B (B_m - B)\right\} \tag{8}$$

The equilibrium point $x_e=0$ is asymptotically stable if:

- V is positive definite (P is positive definite);
- $V(0)=0$;
- dV/dt is negative definite.

The second problem is to obtain the derivative of the function V. [8]

Matrix P is the symmetric and positive definite solution of the Lyapunov equation (8). We assume that Q is the identity matrix.

The derivative of the Lyapunov function V is negative definite if the adaptation laws for the variant closed-loop system matrix are:

$$\begin{cases} A_m^{\ T} \cdot P + P \cdot A_m = -Q < 0 \\ \dot{A}(t) = \gamma_A \cdot P \cdot x_e \cdot \theta^T \\ \dot{B}(t) = \gamma_B \cdot P \cdot x_e \cdot r^T \end{cases} \tag{9}$$

Solving the system (9), from the first equation results the matrix P, relation (10), and then the adaptation laws for the controller parameters, relation (11).

$$P = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix} = \\ = \begin{bmatrix} \frac{1}{2}\cdot\left(\frac{a_m}{b_m} + \frac{1+b_m}{a_m}\right) & \frac{1}{2\cdot b_m} \\ \frac{1}{2\cdot b_m} & \frac{1}{2\cdot a_m}\cdot\left(1+\frac{1}{b_m}\right) \end{bmatrix} \tag{10}$$

$$\dot{k}_1 = \gamma_1\left(p_{12}\cdot\theta_e + p_{22}\cdot\dot{\theta}_e\right)\cdot r \\ \dot{k}_2 = -\gamma_2\left(p_{12}\cdot\theta_e + p_{22}\cdot\dot{\theta}_e\right)\cdot \theta \\ \dot{k}_3 = -\gamma_3\left(p_{12}\cdot\theta_e + p_{22}\cdot\dot{\theta}_e\right)\cdot \dot{\theta} \tag{11}$$

In the adaptation laws (11) some terms were absorbed into the adaptation gains γ_1, γ_2, γ_3.

Figure 3 shows the simulink scheme of the local model reference adaptive control of the robotic arm with D.C. motor gear driven. [7]

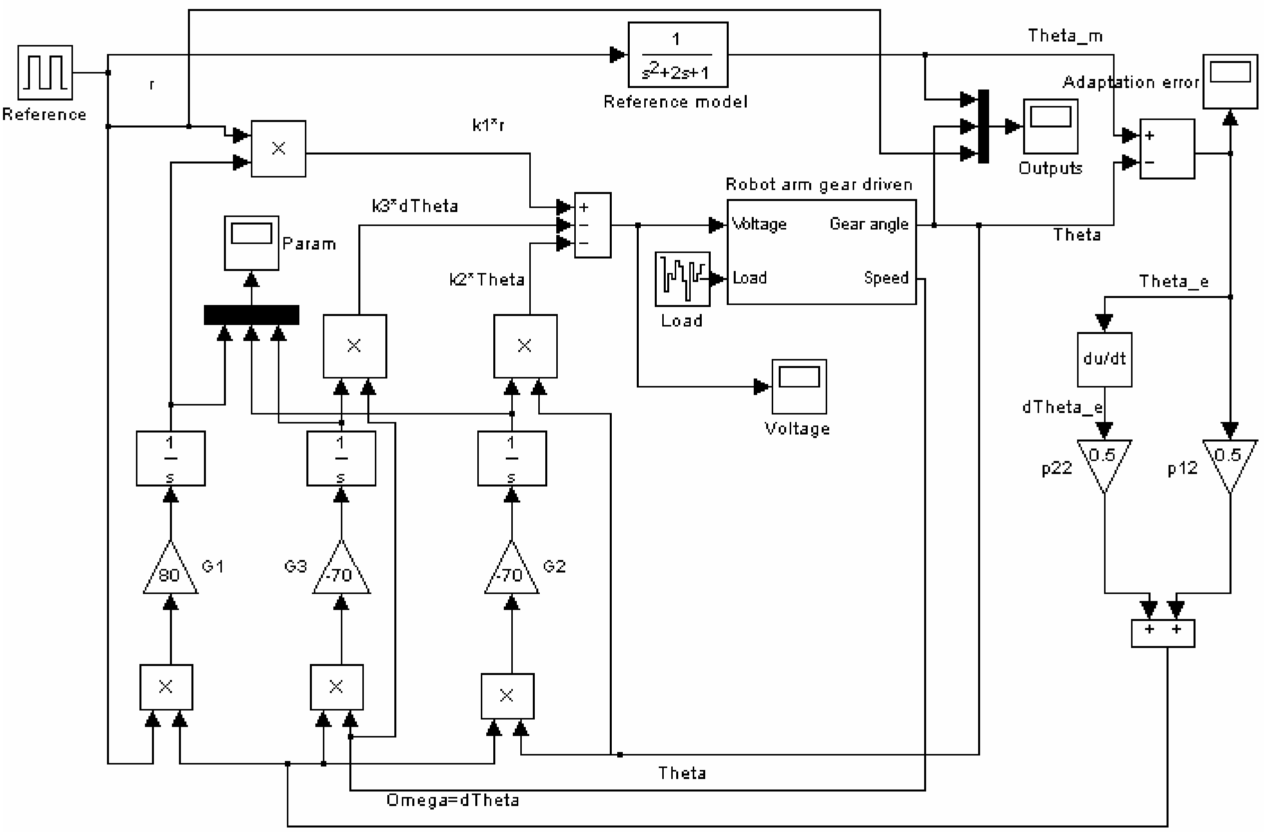

Fig. 3. Local model reference adaptive control of the robotic arm with D.C. motor gear driven.

4. SIMULATION OF THE LOCAL MODEL REFERENCE ADAPTIVE CONTROL OF THE ROBOTIC ARM

This section presents the simulations and results obtained using the control scheme with MRAC structure from figure 3 and the relations previously determined.

In the model design, simulation and system testing the following values were used for physical constants: $\mu=0.01$, $J=8kgm^2$, $k_e=0.5247Vmin/rot$, $k_m=6.1Vmin/rot$, $B=1.5\ Nms/rad$, $R=1.025\Omega$.

The main problem is to test the capability of the closed-loop adaptive system to adapt to a desired trajectory. The first reference proposed for the simulation is a square input with 20 seconds period and $\pi/3$ amplitude.

Although the calculations in the paper were made with the simplified model, the simulation with the real model is similar.

Figure 4 shows the reference and the reference model output (desired evolution of the gear angle).

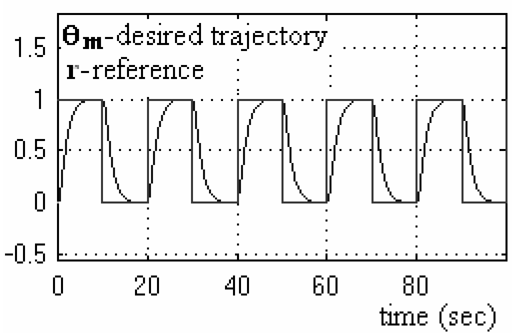

Fig. 4. Reference *r(t)* and desired trajectory for the gear angle $\theta_m(t)$.

The desired trajectory for the robotic arm is a second order response imposed by the model reference system, relations (3) and (4).

Figure 5 shows the reference, the reference model output (desired evolution of the gear angle) and the real gear angle. After two of the impulses the gear angle follows the desired trajectory and so the adaptation error tends to zero. The amplitude of the oscillations from the beginning of the simulation is reduced and practically disappears.

The figure 6 shows the evolution of the adaptive controller parameters.

To test the adaptive capability in different working conditions, the square input was replaced with ramp trajectory between [0, π].

Fig. 7 shows the response of the MRAC system relative to the model response.

5. CONCLUSIONS

The paper studied the possibility to compensate the interdependences between different axes of the robots and unknown loads using a local adaptive control for every arm. So, for robotic arm local control

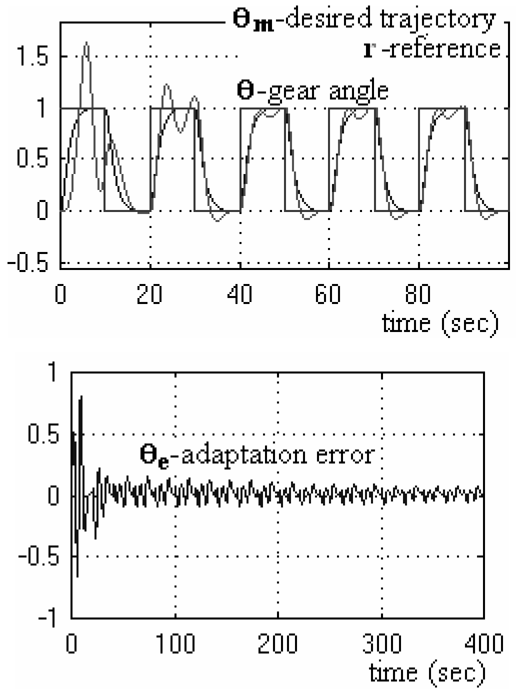

Fig. 5. Reference *r(t)*, desired trajectory for the gear angle $\theta_m(t)$, real gear angle $\theta(t)$ and adaptation error *p(t)*.

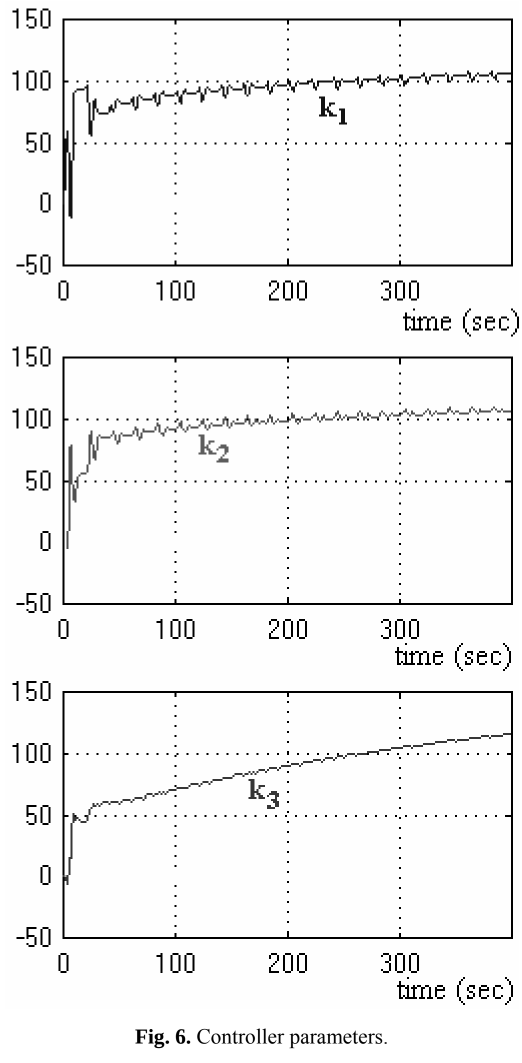

Fig. 6. Controller parameters.

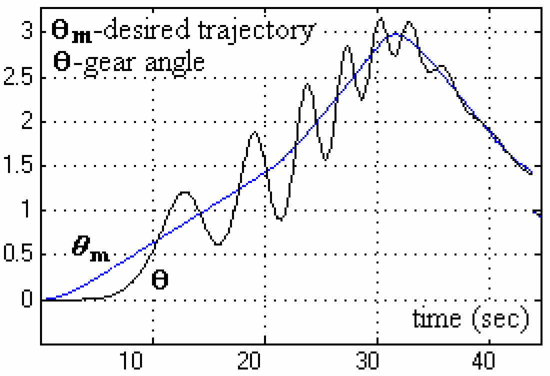

Fig. 7. Desired trajectory for the gear angle $\theta_m(t)$, real gear angle $\theta(t)$.

a model reference adaptive control MRAC was proposed.

Certainly, the studies of the adaptive system behavior may be obtained for many situations. So, for instance, is easy to see qualitative and quantitative difference between responses of the same adaptive system to different prescribed references or to different loads.

Generally, the MRAC scheme applies to systems with known dynamic structure, linear or non-linear, but with unknown constants or slowly varying parameters. The adaptive controller designed for our plant is inherently non-linear. The MRAC system can handle large variations of the plant parameters with slow varying dynamic response. Otherwise, the stability of the closed-loop system and the convergence of the adaptation error are assured by the Lyapunov theory of stability.

Although the robotic arm model used for the simulation is a third order type, for adaptive system design has been used a second order approximate model with similar response. Than, the second order reference model was chosen. Due to the nature of the reference model and plant a PD-type controller was imposed with three parameters. The adaptation laws for these parameters were determined using Lyapunov theory.

The global adaptive system was simulated in Matlab-Simulink to different references. The square and ramp responses (evolution of the gear angle) highlight the capacity of the system to adjust its parameters and to compensate the unknown variations in the system after a specified time

However, there are certain tradeoffs required to achieve this success practically. These include the digital conversion of the adaptive structure, the computational complexity and experiments for the practical implementation.

The method has a great degree of generality and can be easily adapted in study of other control system.

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