Optimal Reactive Power Dispatch and Voltage Control Using Interior Point Method

M. KHIAT, D. REHIEL, A. CHAKER and Z. FRIOUI

***Abstract**: This paper presents an interior point method for optimal reactive power dispatch including equality and inequality nonlinear constraints which represent the power system security conditions. The interior point method used is based on the logarithmic-barrier primal-dual algorithm, for nonlinear programming. The proposed method has been applied to minimize the total active power loss of IEEE-57 bus test systems. Test results indicated that the convergence is facilitated and the number of iterations became smaller.*

Keywords: Load flow, Interior point method, reactive power optimization.

1. INTRODUCTION

Active power loss minimization of electrical transmission power systems (ETPS) is considered a requirement of current competitive electricity markets. In ETPS planning and operation, the security and reliability are assessed using a number of computer programs which include the optimal reactive power dispatch (ORPD). An objective of ORPD problem solution is to determine the optimal steady-state operation of ETPS. Classical methods to solve ORPD include the sequential linear and quadratic programming, reduced gradient and Newton methods [1,2,3,4,5]. Recently it has been used the interior point method (IPM) wits its various alternatives. The IPM has proven the last years to be superior in the presence of case inequality constraints. The IPM has been applied to optimize the power systems operation with peat success solving problems such as the optimal power flow [7,8,9,15,16], VAr dispatch [10,11,17] state estimation, load ability maximization, Voltage stability.

Most of IPM applications in power systems use the primal-dual interior point methods (PD-IPM) despite of the large number of iterations and some divergence problems during the search directions computation by Newton method [15,16,17].

In this paper, a PD-IPM is described to directly solve the large-scale NLP that represents the total real power loss minimization in a power network. Also, several implementation issues are discussed, and an application is made to the IEEE-57 test systems. Results have indicated that convergence is facilitated and the iterations number became smaller.

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2. PROBLEM FORMULATION

The interior point method with its various alternatives has proven the last years to be superior in the case when we impose inequalities constraints.

N. KARMARKAR was a precursor to implementing the first algorithm efficient in both theory and practice. In 1984, he proposed the projective method. This method can be compared to the Simplex algorithm in the experiments on practical problems. Since then, the most famous of the methods of the interior point which is disclosed and studied is the predictive/corrective method**.** This method works well in practice, even if its theoretical study is still imperfect**.** The computer tools have contributed a great deal to the effectiveness of the interior point method with its complex properties. Among these interests we mention the application of the method to the resolution of the problem of the optimal distribution of powers (ODP).

Since the introduction of the interior point method, several articles using this method and its variants to solve optimization problems of power have developed [7, 8, 9, 15].

2.1. Interior Point Algorithm for Non Linear Programming

The mathematical formulation of the optimization problem can be expressed as the following mixed-integer nonlinear programming (MINLP) problem:

$$Min\ f(z)$$

$$\text{Subject to: } g(z) = 0 \qquad (1)$$

$$z_{min} \le z \le z_{max}$$

Where:

$f(z)$ is the objective function

$g(z)$ are equality constraints
z_{min} is the lower limit of variable z
z_{max} is the upper limit of variable z.

The inequality constraints of (1) are transformed in equality constraints by using non-negative slack variables (z_u, z_l). The MINLP problem (1) becomes:

$$\begin{aligned} & Min\ f(z) \\ & \text{Subject to: } g(z) = 0 \\ & z_u + z - z_{max} = 0 \\ & z_l - z + z_{min} = 0 \\ & z_u, z_l \geq 0 \end{aligned} \tag{2}$$

The slack variables are included in $f(z)$ as logarithmic terms (logarithmic-Barrier function), so the MINLP problem (2) becomes:

$$\begin{aligned} & f(z) - \mu \sum \left(lnz_{uj} + lnz_{lj} \right) \\ & \text{Subject to: } g(z) = 0 \\ & z_u + z - z_{max} = 0 \\ & z_l - z + z_{min} = 0 \\ & z_u, z_l \geq 0 \end{aligned} \tag{3}$$

Where the barrier parameter $\mu > 0$ and is decreased to zero as the algorithm iteration progresses.

The Lagrangean function L of (3) is:

$$L = f(z) - \mu \sum \left(lnz_{uj} + lnz_{lj} \right) - \lambda^t g(z) - \gamma^t (z_u + z - z_{max}) - \phi^t (z_l - z + z_{min}) \tag{4}$$

Where $\lambda, \gamma\ and\ \phi$ are Lagrange multipliers vectors.

Based on the Karush-Kuhn-Tucker (KKT) first-order conditions of the sub-problem, a set of nonlinear algebraic equations is formed and then solved by the Newton-Raphson algorithm [4,]. The iteration procedure of the IPM is stopped when the mismatches of KKT conditions are sufficiently small or less than the specified tolerance ε as shown in the following:

$$\|L_z\| = \|\nabla f(z) - \nabla g^t(z)\lambda - \gamma - \phi\| \leq \varepsilon \tag{5}$$

$$\|L_\lambda\| = \|g(z)\| \leq \varepsilon \tag{6}$$

$$\|L_\gamma\| = \|z_u + z - z_{max}\| \leq \varepsilon \tag{7}$$

$$\|L_\phi\| = \|z_l - z + z_{min}\| \leq \varepsilon \tag{8}$$

$$\gamma^t z_u + \phi^t z_l \leq \varepsilon \tag{9}$$

In the primal-dual theory, z, z_u and z_l are the primal variable; λ, γ and ϕ are the dual variables; equation (5) is the dual feasible conditions; Equations (6), (7) and (8) are the primal feasible conditions; and (9) is the complementary slackness condition. Therefore, the optimal solution fulfills the stopping criteria in (5)-(9), white the feasible solution satisfies the stopping criteria in (6)-(8).

3. IPM APPLICATION FOR ORPD

The minimization of the total active power losses through the optimization of generator and compensator reactive powers, and switchable reactive power sources (banks of capacitors, reactors, SVC, etc.) which can be formulated as:

$$Min\ f(v, \theta) \tag{10}$$

The problem is subject to the following constraints:

- Nonlinear equality-constraints (load-flow equations):

$$q(v, \theta, q) = 0 \tag{11}$$

$$p(v, \theta, p) = 0 \tag{12}$$

- Voltage- magnitudes limits at all nodes:

$$v^{min} \leq v \leq v^{max} \tag{13}$$

- Limits on generator and compensator reactive powers:

$$q_g^{min} \leq q_g \leq q_g^{max} \tag{14}$$

- Limits on reactive power sources (shunt elements):

$$q_{sh}^{min} \leq q_{sh} \leq q_{sh}^{max} \tag{15}$$

Introducing slack variables $v_u, v_l, q_{gu}, q_{gl}, q_{shu}, q_{shl}$, that are included in $f(.)$ as logarithmic terms (logarithmic-Barrier function) the problem can be written as:

$$\begin{aligned} Min\ f(v, \theta) \quad & - \mu \sum_{i=1}^{n} (lnv_{ui} + lnv_{li}) \\ & - \mu \sum_{k=1}^{ng} \left(lnq_{guk} + lnq_{glk} \right) \\ & - \mu \sum_{m=1}^{nsh} (lnq_{shum} + lnq_{shlm}) \end{aligned} \tag{16}$$

Subject to:

$$q(v, \theta, q) = 0 \tag{17}$$

$$p(v, \theta, p) = 0 \tag{18}$$

$$v_u - (v^{max} - v) = 0 \tag{19}$$

$$v_l - (v - v^{min}) = 0 \tag{20}$$

$$q_{gu} - \left(q_g^{max} - q_g \right) = 0 \tag{21}$$

$$q_{gl} - \left(q_g - q_g^{min} \right) = 0 \tag{22}$$

$$q_{shu} - (q_{sh}^{max} - q_{sh}) = 0 \tag{23}$$

$$q_{shl} - \left(q_{sh} - q_{sh}^{min} \right) = 0 \tag{24}$$

The Lagrangian of the above optimization problem can be defined as follows:

$$\begin{aligned} L = & f(v, \theta) - \mu \sum_{i=1}^{n} \left(lnv_{ui} + lnv_{li} \right) \\ & - \mu \sum_{k=1}^{ng} \left(lnq_{guk} + lnq_{glk} \right) - \mu \sum_{m=1}^{nsh} \left(lnq_{shum} + lnq_{shlm} \right) \\ & - \lambda^t q(v, \theta, q) - \tau^t p(v, \theta, p) - \gamma_u^t \left(v_u - v^{max} + v \right) \\ & - \gamma_l^t \left(v_l - v + v^{min} \right) - \eta_u^t (q_{gu} - q_g^{max} + q_g) \\ & - \eta_l^t (q_{gl} - q_g + q_g^{min}) - \psi_u^t \left(q_{shu} - q_{sh}^{max} + q_{sh} \right) \\ & - \psi_l^t (q_{shl} - q_{sh} + q_{sh}^{min}) \end{aligned} \tag{25}$$

Where $\lambda^t, \tau^t, \gamma_u^t, \gamma_l^t, \eta_u^t, \eta_l^t, \psi_u^t, \psi_l^t$ are the Lagrange multipliers. The KKT optimality conditions lead to the following matrix equations:

$$\nabla_\theta L = \nabla_\theta f - Q_\theta^t \lambda - P_\theta^t \tau = 0 \quad (26)$$

$$\nabla_v L = \nabla_v f - Q_v^t \lambda - P_v^t \tau - (\gamma_u - \gamma_l) = 0 \quad (27)$$

$$\nabla_{q_g} L = -Q_{q_g}^t \lambda - (\eta_u - \eta_l) = 0 \quad (28)$$

$$\nabla_{q_{sh}} L = -Q_{q_{sh}}^t \lambda - (\psi_u - \psi_l) = 0 \quad (29)$$

$$\nabla_{v_u} L = 0 \Rightarrow S_u \gamma_u = -\mu e \quad (30)$$

$$\nabla_{v_l} L = 0 \Rightarrow S_l \gamma_l = -\mu e \quad (31)$$

$$\nabla_{q_{gu}} L = 0 \Rightarrow Q_{gu} \eta_u = -\mu e \quad (32)$$

$$\nabla_{q_{gl}} L = 0 \Rightarrow Q_{gl} \eta_l = -\mu e \quad (33)$$

$$\nabla_{q_{shu}} L = 0 \Rightarrow Q_{shu} \psi_u = -\mu e \quad (34)$$

$$\nabla_{q_{shl}} L = 0 \Rightarrow Q_{shl} \psi_l = -\mu e \quad (35)$$

The above optimality conditions equations (KKT) are solved by a single-step Newton's method that could involve a predictor-corrector scheme [10,11,15,16,17]. Two approaches are possible:

a) Equation (19-24) and (30-35) are first used to eliminate the slack variables and the corresponding Lagrange multipliers, and then the resulting reduced system is linearized [10,11].
b) The whole system is first linearized , and then the respective equations are used to eliminate those unknowns, yielding a reduced linear system [10,11]. This latter approach is applied in this paper.

The procedure to solve the KKT of the Logarithmic-Barrier Primal Dual Algorithm (LBPDA) problem can be summarized as follows. First, we assume a starting point that satisfies the positivity condition on the slack variables, and a barrier parameter $\mu^{(0)} > 0$ that causes the objective function logarithmic terms to dominate over the value of the original objective $f(v^{(0)}, \theta^{(0)})$. Second, the KKT equations are solved by one iteration of the Newton's method. Third, all the variables are updated. Fourth, the barrier parameter is appropriately reduced to the next point. This iterative process is repeated until primal and dual feasibilities are achieved within acceptable accuracy, and a stopping criterion is satisfied.

4. IMPLEMENTATION AND RESULTS

The proposed method has been implemented. The IEEE 57 bus test systems have been used in the simulations. In Table 1 it is shown relevant information limits for all systems used in the tests. Also, the initial active power loss *(Ploss)* is shown (initial case).

We present different results and analyze the optimal allocation and a comparison between different applications that are organized as follows:

1. **Case 1**: Initial state (base case load-flow).
2. **Case2**: Application of the reduced gradient method after voltages correction.
3. **Case 3**: Implementation of interior point method without correction voltages.

Table 1. Limits.

	MIN	**Max**
$V_1..V_{57}$[p.u.]	0.9	1.1
Q_2 [MVAr]	-17.0	50.0
Q_3 [MVAr]	-10.0	60.0
Q_6 [MVAr]	-8.0	25.0
Q_8 [MVAr]	-140.0	200.0
Q_9 [MVAr]	-15.0	90.0
Q_{12} [MVAr]	-50.0	155.0
Q_{c18} [MVAr]	0.0	20.0
Q_{c25} [MVAr]	0.0	25.0
Q_{c53} [MVAr]	0.0	10.0

1. **Case 1**: initial state (base case load-flow).

In this case, we present the results of the initial state and the profile of voltages (Fig.1). We obtain the following results:

P_{GT}=1279.89 MW; P_{CHT}=1250.80 MW;

Ploss=29.09 MW.

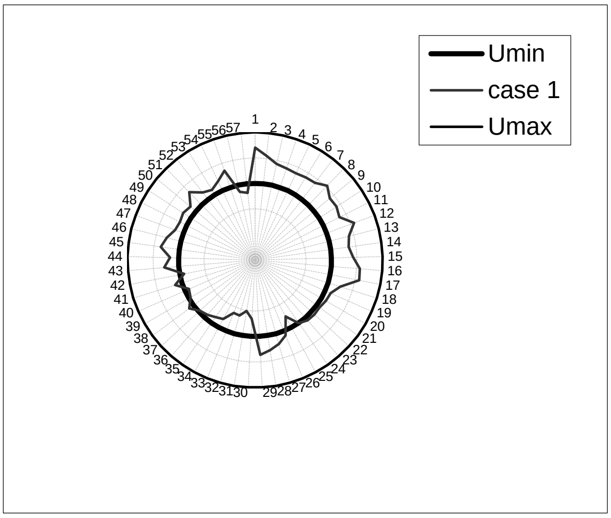

Fig. 1. Bus voltage, case n° 1.

In the initial state, nodes 25, 30, 31, 32, 33, 34, 35, 36, 37, 39, 40, 42, 56, 57 have exceeded the lower limit is imposed of 0.9 could the crux of the greatest violation in this case is node 31. Among all the control devices of the network [4], the first control device is a capacitor placed at node 25. The action is to produce 13 Mvar of reactive power (in this step we process by injecting reactive power by not 1Mvar). The second control device is the processor 56-57. The action is calculated to reduce the transformation ratio of 0.03. The third control device is the processor 11-43. The action is calculated to reduce the transformation ratio of 0.03. The fourth control device is the processor 11-41. The action is calculated to reduce the transformation ratio of 0.04. (The changing relations of transformation have been done in steps of 0.01).

2. **Case 2**: Applying the reduced gradient method after voltages correction.

In this case, we present the results as the gradient reduced after the correction of voltages and stress contours (Fig. 2). We acquire the following results:

P_{GT}=1277.53 MW; P_{CHT}=1250.80 MW;

Ploss=26.73 MW.

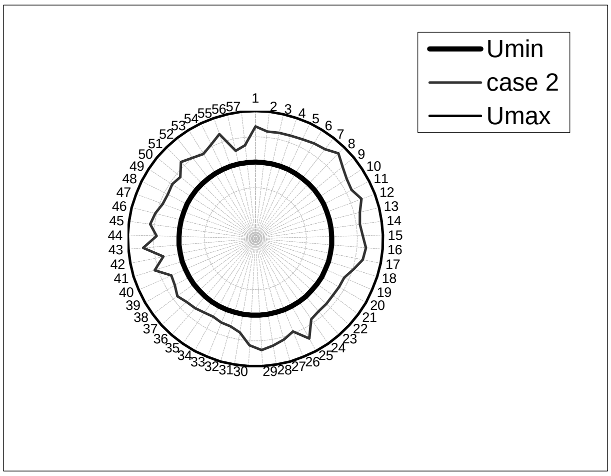

Fig. 2. Bus voltages, case n° 2.

3. **Case 3**: Application of interior point method (technical log-barrier) without correction voltages.

In this case, we present the results as the interior point (technology journal barrier) and the contour of tension (Fig. 3). In case 3, it appears that, without modifying the network, i.e. without changing the controls were a good voltage profile. We acquire the following results:

P_{GT}=1274.18 MW; P_{CHT}=1250.80 MW;

Ploss=23.38 MW.

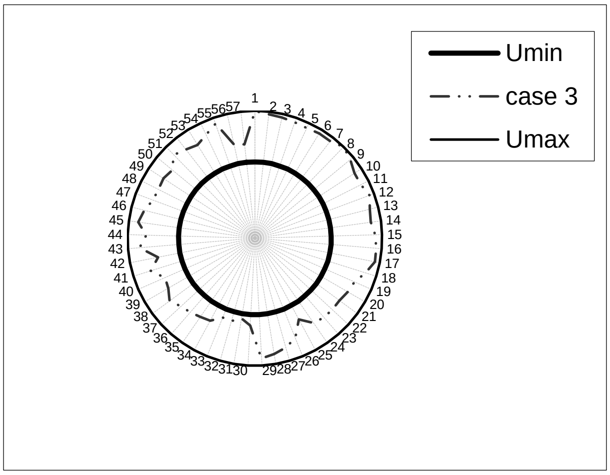

Fig. 3. Bus voltages, case n° 3.

Powers:

According to the results, we find that for all three cases, all the active powers generated are fixed, except that the node balance. The active power of case 3 is the smallest. Based on the three cases we see a big difference between reactive powers generated.

Table 2. Active and reactive powers generated.

Bus	Case 1		Case 2		Case 3	
	P_G [MW]	Q_G [MVAR]	P_G [MW]	Q_G [MVAR]	P_G [MW]	Q_G [MVAR]
1	479.89	142.32	478.05	23.30	474.18	34.62
2	0.00	5.73	0.00	15.0	0.00	50.00
3	40.00	14.05	40.00	33.0	40.00	60.00
6	0.00	8.09	0.00	10.40	0.00	16.28
8	450.00	67.92	450.00	68.00	450.00	39.38
9	0.00	11.39	0.00	48.30	0.00	76.68
12	310.00	139.98	310.00	104.20	310.00	78.55

Table 3. Reactive powers of shunt capacitors.

Bus	Case 1	Case 2	Case 3
18	10.00	10.00	10.00
25	5.90	18.90	5.90
53	6.30	6.30	6.30

Total active power losses:

According to Table 4, we find that the total active power losses cases 3 are 23.38 MW. This result is better compared to the previous case. There is a decrease of 19.63% of losses compared to the case 1.

Table 4. Total active power losses [MW]

Bus	Case 1	Case 2	Case 3
P_{GT}	1279.89	1277.53	1274.18
P_{CHT}	1250.80	1250.80	1250.80
Ploss	29.09	26.73	23.38
Reduction en %		6.32	19.63

5. CONCLUSION

The ORPD by a PD-IPM method has been presented in this paper. The problem formulation has been addressed in detail and several implementation issues haves been discussed. After formulating the original NLP, the Log-Barrier technique has been applied to the resulting linearized system. Simulation and results of applying a prototype version to The IEEE 57 bus are presented and discussed.

The results obtained by the interior point method which is based on the technical log-barrier are very satisfactory and better than those obtained by the reduced gradient method. All voltages were corrected. This demonstrates that the interior point method is very effective in satisfying all inequality constraints. There is a decrease of 19.63% for total active power losses compared to the initial case.

Results of the tests have indicated that the convergence is facilitated and the number of iterations may be small.

Finally the advantages of this method are: optimal solution, flexibility and customized solution. This procedure allow us to determine a strategy that results in keeping the voltage within the required limits as well as in a significant reduction of the total active power losses.

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