Improvement of Sparse Computation
Application in Power System Short Circuit Study

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Abstract: This paper presents a contribution that enhances the efficiency of sparsity techniques for solving some problems in large scale power system. Alterations are introduced on storage scheme and nodes ordering in references [8,9] to led a reduction of the computation time and space memory. The fast sparse vector approach (FSV) is presented and implemented in a short circuit program to compute fault currents and fault bus voltages. It is based on a formulation which computes only the required column of the Z impedance matrix (or some elements of the column) for any particular type of fault. The tests carried out on several power systems ranging from the IEEE 30 bus to a 1084 bus test systems show that the proposed approach gives an important reduction of the computation time.

1. INTRODUCTION

Most of the power network computer applications are formulated as a set of sparse linear equations. The main computational burden, particularly in on-line network calculations, resides in the solution of these network matrix equations. Sparse matrix techniques, whose very purpose is computational efficiency, are the dominant factors in the choice of the methods. The basic order-factorize-solve sparsity techniques based on either the triangular factorization LDLₜ or the bi-factorization LR methods have been in use in power system software for decades now and are still in production use [1,2]. However, many alterations and refining techniques that offer considerable improvements in efficiency in appropriate applications continue to be introduced by researchers. Partial matrix refactorization has been used in cases where only some elements of the system matrix need to be recalculated [3]. Reference [4] developed an heuristic ordering algorithm which minimizes the fill-ins and path lengths in partial refactorization. In some applications, important savings are also available by exploring the advantages of the sparse vector techniques which avoid the unnecessary computations when solving sets of sparse linear equations characterized by a sparse unknown vector and/or a sparse constant vector x/b [5] and using the so called fast-forward and fast-backward substitutions [6]. The fast-forward/fast-backward feature can be further explored to speed up calculations in problems requiring just a single value or a single solution such as in fault studies.
This paper presents a contribution on improving the efficiency of sparse vector techniques by performing a special node ordering to avoid the need of diagonal multiplications and the fast forward substitutions in computing any column of Z matrix. If some elements of the columns are wanted, a part of the backward substitution operations can be also avoided. The approach is named the fast sparse vector (FSV) and is implemented in a short circuit program to compute fault currents (or short circuit capacities) and fault bus voltages. In the following section a brief review of the short circuit equations is given [7,10].

2. SHORT CIRCUIT STUDY

Short circuit studies and fault analysis are therefore very important since they provide data such as voltages and currents during and after the different types of faults which are necessary in designing and monitoring the protective schemes of the power system in both planning and operation stages.

The usual short circuit computations are concerned with three phase faults and single phase-to-ground faults as they are considered to be the severest and the most frequent. Supposing the fault location to be at a real bus marked q and the prefault bus voltage is 1.0 p.u., then the following short circuit equations are obtained by using Thevenin’s theorem (Figure 1):

\[ I_{qf}^f = \frac{1}{Z_{qq}} \]  \hspace{1cm} (1)

\[ V_{i}^f = 1 - \frac{Z_{qi}}{Z_{qq}} \quad i = 1, \ldots, n \]  \hspace{1cm} (2)

where \( n \) is the number of buses, \( q \) the faulted bus, \( I_{qf}^f \) the fault current (or the fault level), \( V_{i}^f \) the fault voltage of bus \( i \), \( Z_{qi} \) the impedance matrix diagonal element corresponding to the faulted bus and \( Z_{qq} \) the impedance matrix element located in line \( i \) and column \( q \). The fault current in a transmission line \( ij \) is given by:

\[ I_{ijf}^f = \left[ V_{i}^f - V_{j}^f \right] y_{ij} \]  \hspace{1cm} (3)

where \( y_{ij} \) is the admittance of the line connected between the buses \( i \) and \( j \).

For each fault on a bus location \( q \), the short circuit current and the fault bus voltages are calculated using the above equations. It should be noted here that for each fault at bus location \( q \), the corresponding \( q \)th column element of the \( Z \) impedance matrix needs to be computed by inverting the \( Y \) network admittance matrix.

In the short circuit study it is sometimes necessary to know currents (or short circuit capacities) of lines connected to the faulted bus. The nodes connected to \( q \) are called \( M_i \). The three-phase short-circuit capacity of the faulted node \( q \), \( S_{qq} \) and the short-circuit capacity \( S_{Mi,q} \) in the related branch \( M_i-q \) are given by the following equations:

\[ S_{qq} = \frac{MVA}{|Z_{qq}|} \]  \hspace{1cm} (4)

\[ S_{Mi,q} = S_{qq} \left| \left( Z_{qq} - Z_{Mi,q} \right) Y_{Mi,q} \right| \]

\( S_{qq} \) and \( S_{Mi,q} \) are expressed in MVA and \( MVA \) is the system power reference.

It should be noticed, however, that not all the elements of \( q \) column are needed, only those which correspond to the same position of non-zero elements of the \( Y \)-matrix. Thus the FSV approach presents an efficient technique in order to calculate only the necessary elements \( Z_{qq} \) and \( Z_{Mi,q} \).

It appears then that the main computation burden resides in the storage and inversion of the network admittance matrix. To overcome this drawback, next sections present a sparse storage scheme for the \( Y \) matrix and its factor.
matrices. The fast sparse vector approach uses this sparsity based matrix storage scheme to compute only the required column of the Z impedance matrix for any fault location by performing a special node ordering and avoiding the usual solution steps.

3. STORAGE SCHEME

In order to exploit the benefits of the symmetry and the sparse nature of the power network Y admittance matrix, a pact matrix storage scheme in which only the non-zero elements are retained is employed. For this purpose, it is necessary to introduce tables of indexing information to identify the non-zero elements and to facilitate their addressing. The storage strategy used is the so-called linked lists, which make provision for the structural modifications which are expected as a result of the matrix processing [8]. It consists of two parts: a complex vector to hold the numerical values of the admittance matrix elements and an overhead storage comprising three integer vectors used as pointers. In this scheme, only the non-zero elements are stored. The same vectors are used to store the factor matrices produced by the factorisation process of the original matrix. Considering n as the number of network buses and nz the number of non-zero elements and non-diagonal of the Y matrix. The used vectors in the storage scheme are defined as follows:

- **LCOL(n)** is a vector of n elements giving the element number of the first non-zero element in each column.
- **NOZE(n)** is a vector of n elements representing the number of the non-zero elements and non-diagonal in each column.
- **ITAG(nz)** is the vector of nz elements pointing to the row numbers of the non-zero elements.
- **LNXT(nz)** is the vector of nz elements giving the column number of the next non-zero element. If the element is the last one in the column, the corresponding LNXT entree is equal to zero.
- **CE(nz)** is a vector containing the numerical values of non-zero element and non-diagonal.
- **D(n)** is a vector containing the numerical values of diagonal elements.

In order to reduce the space memory, the diagonal elements are stored in D vector, because each element needs only one index and the D vector can be transformed in a solution vector.

4. SPARSE COMPUTATION

The Z matrix is usually obtained by inverting the Y matrix. To compute any particular Z matrix column q required for short circuit studies, reference [9] applied the sparse matrix techniques to solve the network set of equation \( Yz = I \) by exploring the sparse vector feature of the constant vector I and thus performing a fast forward and a fast backward substitutions. The Y matrix is factorized to the \( LDL^T \) form, \( z \) is the solution vector containing the \( q^{th} \) column of the Z matrix and I is a singleton vector having the \( q^{th} \) element equal to 1 and the remaining elements are zero.

This paper is an enhancement to the above work leading to a faster solution which generates the relevant Z matrix elements required for any fault location. The \( Yz = I \) set of equations are solved by performing a special node ordering, avoiding the forward substitution and keeping only few backward substitution operations. To illustrate this FSV approach, let the set equations \( Yz = I \) be written in terms of the factorized form as:

\[
LDL^T z = I \quad (5)
\]

The solution of (5) is given by

\[
z = (L^T)^{-1}D^{-1}L^{-1} I
\]

It can be obtained in three steps as follows:

\[z = L^{-1}I\]
\[D = D^{-1}z\]
\[z = (L^T)^{-1}D\]

(6)
(6) symbolizes the fast forward substitution, the diagonal multiplication and the fast backward substitution respectively.

If during the symbolic factorization stage, the faulted bus q is put in the last position that is q = n, then the singleton I becomes:

\[ I = \{0,0,\cdots,0,1\}^T \]

and the above solution steps (6) reduces to

\[ z = I \]

\[ D = D^{-1}I = \{0,0,\cdots,0,d_n\}^T \]  

\[ z = (I')^{-1}D \]  

\[ d_n \] is the n\textsuperscript{th} element of the diagonal matrix factor D.

It is clear from the above equations that the determination of the unique non-zero element of the two singletons z' and D' is straightforward and does not require any computation effort to be made. Therefore the fast forward substitution and the diagonal multiplication computations are totally eliminated.

If only the fault current at a bus n is required in the short circuit analysis, then only the diagonal element \( Z_{nn} \) needs to be computed and therefore, the three solution steps (6) reduce to a single arithmetic operation:

\[ Z_{nn} = 1/d_n \]  

To calculate some elements of the columns we use the Fast Back Substitution. If the number m of the elements \( Z_{iq} \) to be calculated is smaller than n, we classify the nodes i in order to have the last positions before q, the number of operations (6) will be minimized to \( m(m+1)/2 \) multiplications and \( m(m-1)/2 \) additions.

In the case of computing the short-circuit capacities of the related branches of the faulted node, the operation number will be 18 in maximum (for 4 nodes connected to the faulted bus which is a maximum for a classical network configuration) and it is independent of the size of the network.

Therefore, the proposed faster sparse vector solution which generates the relevant Z matrix elements required for any fault location is obtained by using the last equation of (6) and in some cases only few operations are used.

5. TEST RESULTS

The FSV approach has been implemented into a short circuit program written in Fortran language under Windows 98 and has been used to compute fault currents, fault bus voltages and short circuit capacities of the related lines of faulted node as expressed by equations (1), (2) and (4). The program is run on a Pentium II, 200MHz.

To evaluate the performances of the proposed FSV approach, series of tests have been carried out on several power systems ranging from the IEEE 30 bus test system to a 1084 bus test system. The main characteristics of these systems are shown in Table 1.

<table>
<thead>
<tr>
<th>Test system</th>
<th>Buses</th>
<th>Branches</th>
<th>Generators</th>
</tr>
</thead>
<tbody>
<tr>
<td>IEEE 30-bus</td>
<td>30</td>
<td>43</td>
<td>6</td>
</tr>
<tr>
<td>59-bus</td>
<td>59</td>
<td>178</td>
<td>10</td>
</tr>
<tr>
<td>IEEE 118-bus</td>
<td>118</td>
<td>176</td>
<td>53</td>
</tr>
<tr>
<td>685-bus</td>
<td>685</td>
<td>1301</td>
<td>90</td>
</tr>
<tr>
<td>985-bus</td>
<td>985</td>
<td>1280</td>
<td>135</td>
</tr>
<tr>
<td>1084-bus</td>
<td>1084</td>
<td>1441</td>
<td>167</td>
</tr>
</tbody>
</table>

Table 2 shows the operations counts of the proposed fast sparse vector approach (FSV) compared to those of the sparse vector short circuit program (SV) of reference [9]. The operations considered are the multiplications and additions required to compute a Z matrix column corresponding to a given faulted bus.

<table>
<thead>
<tr>
<th>Test system</th>
<th>SV methods</th>
<th>FSV methods</th>
<th>Ratio FSV/SV</th>
</tr>
</thead>
<tbody>
<tr>
<td>IEEE 30-bus</td>
<td>140</td>
<td>55</td>
<td>0.375</td>
</tr>
<tr>
<td>59-bus</td>
<td>277</td>
<td>109</td>
<td>0.393</td>
</tr>
<tr>
<td>IEEE 118-bus</td>
<td>648</td>
<td>265</td>
<td>0.401</td>
</tr>
<tr>
<td>685-bus</td>
<td>4415</td>
<td>11865</td>
<td>0.422</td>
</tr>
</tbody>
</table>
Figure 2 shows the computing times as a function of the size of the test systems for computing a Z matrix column corresponding to a given fault location using the FSV and SV methods. The computing time considered here is the time needed for each method to construct the network Y matrix, computing its L and D factor matrices, computing the Z matrix column and calculating the fault current and the fault bus voltages for a given fault location.

Analysis of table 2 shows clearly that the ratio of the number of operations of the proposed FSV approach to that of the SV method is around 0.4 for all the test systems. This means that for computing the Z matrix column for a fault at a given bus, 60% of the computational effort is earned by the use of the FSV approach.

Figure 2 demonstrates the superiority and the potential of the proposed FSV approach illustrated by the important reduction in the computing time due to the elimination of all fast forward substitutions, the diagonal multiplications and part of the fast backward substitution operations. It should be noted that if the fault bus voltages are not needed then the three solution steps are eliminated and the fault current or the fault level of the corresponding faulted bus is calculated by a simple use of equations (1) and (8).

6. CONCLUSIONS

Refined solutions of power systems sparse linear equations using sparse vector techniques have been the main focus of this paper. A fast sparse vector approach named FSV has been formulated and tested for computing the relevant elements of the Z matrix required to determine the fault current or the fault level and the fault bus voltages of a power system network. It has been shown that the proposed FSV approach has the following properties:
- Reduction by 60% of the computational effort compared to the sparse vector based short circuit program SV.
- Reduction of the space memory by using sparsity techniques based on a specific storage scheme and on the use of dynamical management.
- Avoiding the complex algorithms induced by the fast forward and fast backward substitution.
- Limiting to a maximum of 18 operations independently of the size of the network, for the short circuit capacities of faulted node and related lines.

Further work would be devoted to developing similar sparse refined solutions of other power system problems such as transient stability.

REFERENCES


