Stepper Motor Model for Dynamic Simulation

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Abstract: *The modeling and simulation of the electromechanical behaviour of stepper motors are of high interest because they are often used for space application. A commercially available multibody dynamic simulation software, which sets up the equations of motion and the transfer function of the motor and solves them numerically, was used. The results show, that both the dynamic simulation software and the simulation model are well suited to predict the behaviour of drive units equipped with a stepper motor.*
The model allows microstepping and failure cases to be simulated.

1. Introduction

Stepper motors are simple, robust and reliable and are well suited for open or close loop controlled actuators[1][3].

Steppers can be found in machine tools, typewriters, printers, watches, etc. In space applications stepper motors are mainly used as actuators of pointing mechanisms for antennas, mirrors, telescopes or complete payloads. In order to investigate the dynamics of mechanisms driven by stepper motors a model had to be created.

The electromechanical behavior of a stepper motor was described by control loops. This software calculates the dynamics of rigid and flexible bodies including the control loop dynamics. It generates the equations of motion and the transfer function numerically. A principal stepper motor control loop is given in figure 1.

There are basically 3 types of stepper motors:

- permanent magnet (PM)
- variable reluctance (VR)
- hybrid

All types are considered.
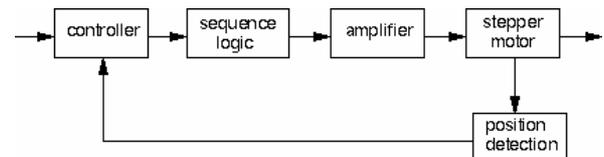
Fig. 1. Stepper motor control system.

Stepper motors are usually used in connection with gears, hence a gear model as well as electrical and mechanical stiffness, friction and resistance are considered [4]. For the purpose of this simulation a model of a 2 phase motor is build up.

An option to extend the model to a 3 or 4-phase stepper will be discussed later. To simplify the model a 90 degrees step size is discussed here.

2. Model generation

2.1. Basic equations

The principle of a 2-phase stepper motor is given in figure 2.

The rotor is a permanent magnet consisting of 1 pole pair. When the windings of one phase are energized, a magnetic dipole is generated on the stator side. If for example phase 2 is active (Phase 1 is switched off),
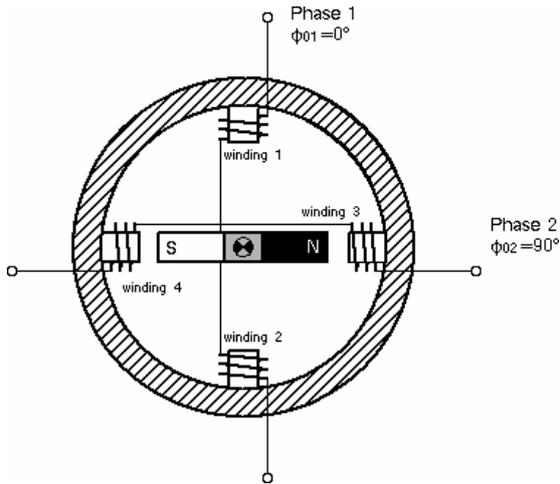
Fig. 2. Principle of a 2 phase stepper motor.

winding 3 produces an electrical south pole and winding 4 an electrical north pole. Figure 2 shows the rotor in a stable position with phase 2 only powered.

The number of steps per revolution of the rotor is calculated as:

$$S = 2 \cdot n \cdot m \tag{1}$$

where n is the number of rotor pole pairs and m the number of stator phases. For the hybrid stepper motor, n is half the number of rotor teeth.

The stepping angle is:

$$\Delta\phi = \frac{360}{s} \tag{2}$$

For the example of we have n=1 and m=2. So that we have 4 steps per revolution and a stepping angle of 90 degrees, what is obvious from figure 2. If a sinusoidal characteristic of the magnetic field in the air gap is assumed, the contribution of each phase j on the motor torque T_{Mj} can be written as:

$$T_{Mj} = k_m \cdot sin[n\phi(t) + \phi_{0j}] \cdot I_j(t) \tag{3}$$

where:

- k_m is the motor constant, depending on the design of the motor
- $\phi(t)$ is the actual rotor position
- ϕ_{0j} is the location of the coil j in the stator
- $I_j(t)$ is the current in the coil as function of time.

The current $I_j(t)$ in the coil is a function of the supplied voltage U_j and the coil properties. A general equation between U_j and $I_j(t)$ is given by:

$$U_j = emf_j + R \cdot I(t) + L\frac{dI(t)}{dt} \tag{4}$$

where:

- emf_j the electromotive force induced in the phase j
- R the resistance of the coils
- L the inductance of the coils.

The emf in each coil can be expressed as:

$$emf_j = k_m \cdot sin[n\phi(t) + \phi_{0j}] \cdot \omega \tag{5}$$

where ω is the rotational velocity of the rotor.

Resistance and inductance of all coils in the motor are the same so that no indices are required. The differential equation can be expressed in the LAPLACE domain:

$$I = \frac{U}{Ls + R} \tag{6}$$

The total torque produced by the stepper is:

$$T_M = \sum_{j=1}^{m} T_{Mj} \tag{7}$$

Considering the equation of motion of the stepper motor

$$\sum_{j=1}^{m} T_{Mj} = J\frac{d\omega}{dt} + D\omega + T_F \tag{8}$$

where:

- J the inertia of the rotor and the load
- D the viscous damping constant
- T_F frictional load torque

The total load acceleration T_A torque is:

$$T_A = \sum_{j=1}^{m} T_{Mj} - D\omega - T_F \tag{9}$$

For more than 1 phase the arrangement of the coils has to be considered. The location of the coils ϕ_{0j} in the stator as a function of the number of phases m is given in table 1.

Table 1.

No of phases m	2	3	4
ϕ_0 of phase 1	0	0	0
ϕ_0 of phase 2	90	60	45
ϕ_0 of phase 3	-	120	90
ϕ_0 of phase 4	-	-	135

If the sinusoidal characteristic of the torque angle curve is dropped, this model can be used to simulate other types of machines, eg. brushless DC motors.

2.2. Generation of pulses

The pulse generator of the motor produces step commands according to the supplied voltage. It is obvious that for any desired rotor sequence of positions a certain pattern of voltages for phase 1 and 2 as a function of time has to be provided. Therefore it is necessary to know the actual position. This is done in 2 different ways:

1. "detect" the rotor position by an encoder, or,
2. count the number of pulses in each direction.

The first option uses in closed loop control and requires an additional device. Therefore the second will be considered. If count the number of pulses is counted, initially information about the position is also available. If x steps have been taken in positive and y steps in negative direction the actual rotor position ϕ_i is:

$$\phi_i = (x-y) \cdot \Delta\phi \qquad (10)$$

The commanded position ϕ_{i+1} will then be:

$$\phi_{i+1} = \phi_i \pm \Delta\phi \qquad (11)$$

(+) stands for positive, (-) for negative voltage.

The pulses for both phases required for 1 revolution are given in table2.

Table 2.

Rotor position	Voltage in phase 1	Voltage in phase 2
0/360°	0	$-U_2$
90°	U_1	0
180°	0	U_2
270°	$-U_1$	0

The required pulse for each phase with a given ϕ_{i+1} may be calculated by:

$$P_j = sin(\phi_{i+1}n + \phi_0) \qquad (12)$$

The time dependency of the pulse sequence is described by a time increment t:

$$\Delta t = \frac{|fUt|}{m} \qquad (13)$$

where f is a scaling factor and can be used to tune the system. For every time step t one pulse of the related phase is produced. If we furthermore consider the gear ratio k_g of the reduction unit that amplifies the torque by a gain, the control loop of a 2-phase stepper motor based on the eq. is given in figure 3.

A voltage U representing the required rotational motor speed is commanded to block "pulse generator". The voltages U_1 & U_2 are fed to coil 1 and 2 after being processed in the pulse generator. These voltages are decreased by the amount of the induced emf's create the currents I_1 & I_2 in the windings. The currents are transferred into torques via the position dependent motor constants. In the first approach a sinusoidal characteristic has been assumed. For simulating other shapes of the torque angle curve this block has to be modified.
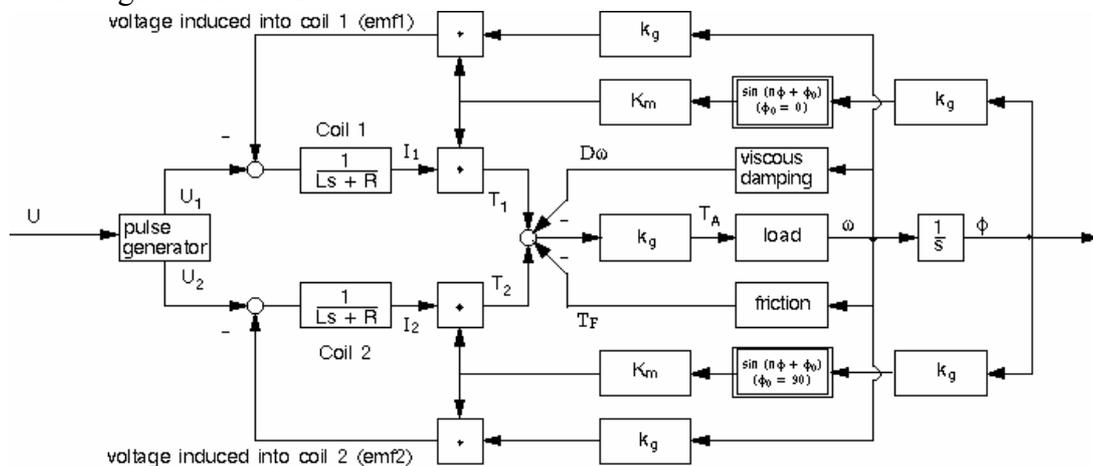
Fig. 3. 2-phase stepper motor (open loop).

The load acceleration torque is the summation of the torques produced by each phase decreased by the resistive torques due to friction and viscous damping. The action of the reduction unit (gearing) is realized by a gain, which amplifies the torque. The load velocity and position after being reduced to the rotor side is used to calculate the emf in the coils.

The element named "load" can be any rotating structure with an inertia. To extend the model to a 3 or 4 phase stepper, a loop 3 and 4 have to be added. In order to verify the model, a test run has been performed.

The data of a typical hybrid stepper motor (23 PP series) is given in table 3.

Table 3.

Operating voltage U	27 V
Static holding torque T_M	0.5 Nm
Winding resistance R	100 ohms
Electrical time constant $_E$	5 ms
Stepping angle	1.8 deg
Number of pole pairs n	50
Load inertia J	1.0 kgm^2

The shaft rotation angle as a function of time and the commanded voltage is given in figure 4. The pulse sequence is indicated additionally.
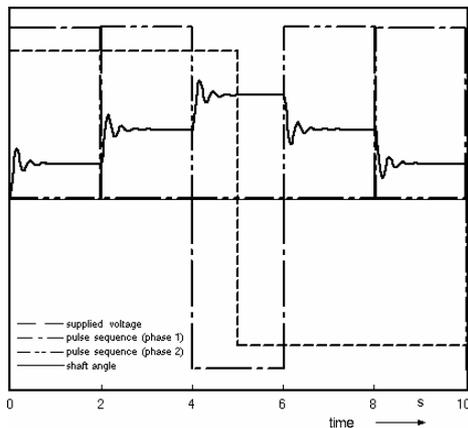

Fig. 4. Stepper motor in open loop.

Each change in angular position shows the oscillatory transient behavior typical for a stepper motor.

3. Stepper motor in closed loop

To run a motor in a closed loop, several additional elements are needed. These are the controller, which evaluates the deviation from the commanded position and the encoder, which detects the position of the rotor. Mathematically the controller can be described as a first order differential equation, whereas the encoder is not needed in the control loop, because the position of the rotor is a primary result of the simulation. Figure 5 shows the control loop of a stepper motor running in a closed loop.
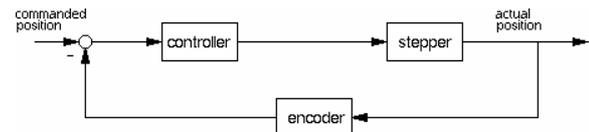
Fig. 5. Stepper motor model closed loop.

In order to interpret the stepper motor model running in closed loop mode a test run has been performed (data according to table 3).

Figure 6 shows the result of the simulation of the closed loop for an ramp input.
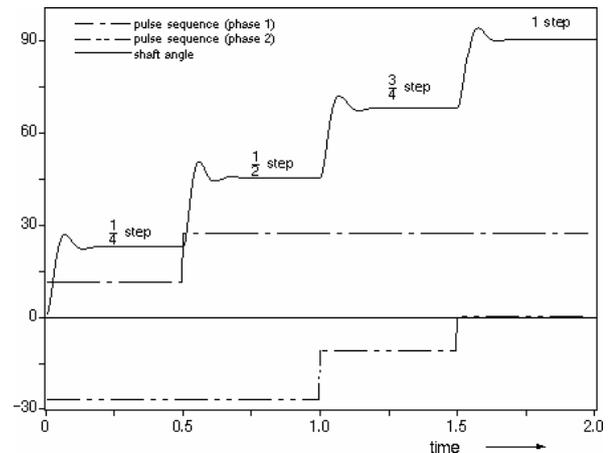
Fig. 6. Commanded and actual position.

When a certain deviation of the actual position of the commanded position is detected, the controller produces a certain magnitude of voltage, which is sufficient to drive the motor following the commanded signal. The higher the deviation from the commanded position, the higher the driving voltage.

4. Simulation of microstepping

There are some applications, where a very fine resolution is required, e.g. in

pointing of antennas. The stepping angle could be reduced by increasing the number of pole pairs (or rotor teeth) or number of phases, which is limited by the manufacturing capabilities. If the step angle must be smaller than about 0.3 deg., a technique called microstepping is used. Microstepping is an operation mode of the motor with 2 phases energized simultaneously. The position of the rotor between 2 adjunct coils is determined by the ratio of the phase currents U_j. A ratio of 1 corresponds hereby to the midposition. Any other ratios lead to other positions. In practice the current of one phase is kept constant over half of the complete step in order to maximize the motor torque.

If U_0 is the maximum available voltage produced by the pulse generator, the required pulse sequence for a subdivision of a step into 4 micro steps is given in table 4.

Table 4.

Step position (angle in deg)	U_1	U_2
0 (0)	U_0	0
1/4 (22.5)	U_0	$k_2 U_0$
1/2 (45)	U_0	U_0
3/4 (67.5)	$k_1 U_0$	U_0
1 (90)	0	U_0

k_j is a predetermined fraction of the current, which is sufficient to rotate the rotor to the required position by ¼ of a step. Considering the assumed sinusoidal torque angle curve, the relationship for the calculation of k is given by:

$$\frac{U_2}{U_1} = tan(n\phi) \qquad (14)$$

to achieve equidistant micro steps. Therefore is:

$$k_1 = \frac{1}{tan(n\phi)} = ctg(n\phi) \qquad (15)$$

and

$$k2 = tan(n\phi) \qquad (16)$$

For the assumed step angle of 90/4 = 22.5 deg, k_2 is 0.4142 for phase 2. This means that the current of phase 1 has to be:

$U_1 = U_0$ and of phase 2 , $U_2 = 0.4142 U_0$

to achieve a micro step of 1/4 of the step size

to position 22.5 degrees.

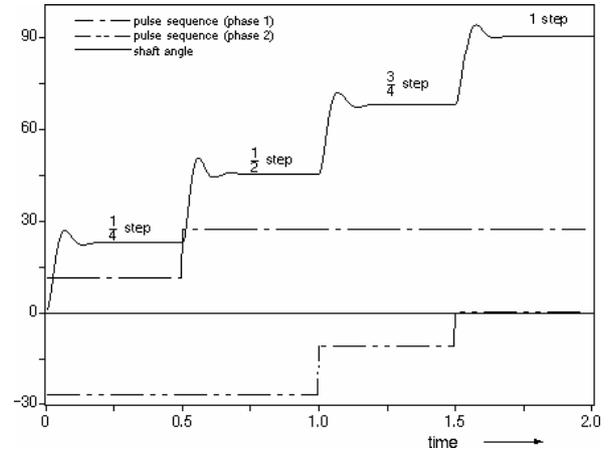The results of a test run performed to check the algorithm is shown in figure 7.

Fig. 7. Stepper motor in open loop (microstepping).

The model provides accurate micro steps of exact 22.5 degrees steps.

5. Failure mode simulation

It has been demonstrated in [4], that the investigation of failure cases is very important to improve the reliability of the stepper performance. In the normal operation mode the rotor is follows accurately the commanded position. However, if the pulse frequency is near to or higher than the natural frequency of the motor, the rotor will loose synchronization.

To investigate this failure mode a simulation was performed. The commanded velocity was increased steadily (see figure 8).
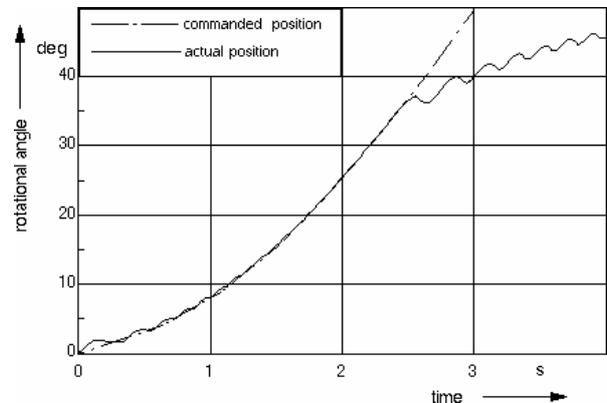

Fig. 8. Failure case simulation.

The motor starts to accelerate until it is running in resonance. After approximately 2.5 sec. the rotor has lost the synchronization

and the behavior becomes unpredictable. Simulations of this kind are very important to investigate and design drive units running under extreme conditions.

6. Conclusions

To investigate and predict the dynamic behavior of mechanisms driven by a stepper motor, a model has been created that describes the electromechanical properties of this drive. To set up the equations of motion of the load and the transfer function of the stepper and its control loop a commercially available dynamic simulation software package has been used.

The created model is qualified to simulate:

- open and closed loop control
- microstepping mode
- failure cases.

By varying the input parameter, stepper motor and control loop design work is possible. The model allows the simulation of all types of stepper motors (VR, PM, hybrid). The results show, that both the software and the model provide realistic statements about stepper motor and load behavior. The model is reliable and numerically stable. This model can also be used for other dynamic simulation programs with only small changes. A future task is to refine the model to consider effects like backlash in the hinge of the gearing. Other drives can be simulated with this model with only small changes, eg. DC motors.

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