ENERGY BASED CONTROL FOR THE WALKING OF A 7-DOF BIPED ROBOT

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Abstract - The focus of this work is to steer a walking motion of a kneed legged biped robot on a level ground, imitating the nearly passive walking on a given slope. Thus we will present some theoretical and simulation results based on the use of an energy based control (referred to as Controlled Limit Cycles).

I. INTRODUCTION

When dealing with walking or hopping biped robots, one of the main objectives is to stand for stable periodic trajectories during regular displacements. From a state space point of view, such objective may correspond to achieve stable limit cycles (e.g. [1] [2] [3]). In such case, the major problem is to look for these potential limit cycles (i.e. to prove their existence), and, if existing, to stabilize the involved periodic or quasi-periodic trajectories (within a chosen state subspace of the biped state space).

A brief survey of existing works points out that several approaches have been investigated to solve such problem. For instance, bipedal walking might be largely understood as a passive mechanical process, as shown for part by McGeer [5], Mochon and MacMahon [4] respectively. Indeed, McGeer [5] demonstrated by both computer simulations and experimental applications, that some legged systems can walk on a range of shallow slopes with no actuation and no control (energy lost in friction and impact is recovered from gravity). Since then, many researchers have considered this passivity based approach (e.g. Goswami et al. [6], Coleman, Garcia et al. [7] Mark.W.Spong [8], F.Asano [9] et al. M.Haruna [10], and reference therein).

However, to our knowledge, none have found a method to define initial conditions under which passive dynamic walking in a downhill slope is generated for an under actuated biped robot with knees and torso. This motivates the main part of the present work which considers the walking motion of a kneed legged biped robot on level ground, imitating the passive walking on a given slope (e.g. [11]).

The paper will be organized as follows:

First we will present the modelling of the 7-dof biped robot under consideration (that is a kneed robot with torso). Then, we will focus on the study of the passive dynamic walking of this robot, on inclined slopes.

Moreover, we will show that before locked both knees and under a simple PD control applied to the lonely actuated link (link between the torso and the stance leg), trajectories of the biped robot can converge towards stable limit cycles. In this context, we will present some results based on Poincaré map method and trajectory sensitivity analysis to efficiently characterize the stability of the almost-passive limit cycles [11].

However, as such limit cycles may not exist for all ground configurations, a complementary control schemes is required. Thus, we will present some theoretical and simulation results based on the use of a recent control method (referred to as Controlled Limit Cycle [1] [2] [3]), which considers the system energy for both controller design and system stabilization.

Finally, some potential extensions for future works will be discussed.

II. THE MODEL

A. Description

The dynamic model of a planar biped robot is considered in this section. It’s shown in figure (1). The
robot has seven degrees of freedom (the five joint angles plus the Cartesian coordinates of the hips, for example). It consists of a torso, hips, and two rigid legs, with knees and no ankles, connected by a frictionless hinge at the hip. This linked mechanism moves on a rigid ramp of slope $\gamma$. During locomotion, when the swing leg contacts the ground (ramp surface) at heel strike, it has a plastic (no slip, no bounce) collision and its velocity jumps to zero. The motion of the model is governed by the laws of classical rigid body mechanics. Following McGeer, we make the non physical assumption that the swing foot can briefly passes through the walk surface when the stance leg is near vertical. This concession is made to avoid the inevitable scuffing problems of stiff-legged walkers like the model analyzed in this paper. It’s assumed that walking cycle takes place in the sagittal plane and the different phases of walking consist of successive phases of single support. With respect to this assumption the dynamic model of the biped robot consists of two parts: the differential equations describing the dynamic of the robot during the swing phase, and the algebraic equations for the impact (the contact with the ground).

![Figure 1](image)

**Figure 1** The model of a 7-dof biped robot downhill a slope

### B. Swing phase model

During the swing phase the robot is described by differential equations written in the state space as follows:

$$\dot{x} = f(x) + g(x)u$$  \hspace{1cm} (1)

With:

$$f(x) + g(x)u = \frac{d}{dt} \begin{bmatrix} \dot{q} \\ \dot{\dot{q}} \end{bmatrix} = \begin{bmatrix} \dot{q} \\ M^{-1}(q)(-H(q, \dot{q}) + Bu) \end{bmatrix}$$

Where $x = (q, \dot{q})$.

(1) Is derived from the dynamic equation between successive impacts given by:

$$M(q)\ddot{q} + H(q, \dot{q}) = Bu$$  \hspace{1cm} (2)

Where

$$q = (q_{31}, q_{32}, q_{41}, q_{42}, q_t),$$

$$u = (u_1, u_2, u_3, u_4),$$

$$M(q) = [5 \times 5]$$ is the inertia matrix and $H(q, \dot{q}) = [5 \times 1]$ is the coriolis and gravity term (i.e.: $H(q) = C(q, \dot{q}) + G(q)$) while $B$ is a constant matrix. The matrices $M, C, G, B$ are developed in [21].

### C. Impact model

The impact between the swing leg and the ground (ramp surface) is modelled as a contact between two rigid bodies. The model used here is from [18], which is detailed by Grizzle & al. in [16]. The collision occurs when the following geometric condition is met.

$$x_2 = z_2 \tan \gamma$$  \hspace{1cm} (3)

Yet, from biped’s behaviour, there is a sudden exchange in the role of the swing and stance side members. The overall effect of the impact and switching can be written as:

$$h: S \rightarrow \chi$$

$$x^+ = h(x^-)$$  \hspace{1cm} (5)

Where $S = \{(q, \dot{q}) \in \chi / x_2 - z_2 \tan \gamma = 0\}$, with $h$ is specified in [21]. The superscripts (-) and (+) denote quantities immediately before and after impact, respectively.

### D. Overall model

The overall 7-dof biped robot model is written as follows:

$$\begin{cases} \dot{x} = f(x) + g(x)u \\ x^+ = h(x^-) \end{cases} \quad x(t) \in S, \quad x^-(t) \notin S$$  \hspace{1cm} (6)

### III. ALMOST PASSIVE DYNAMIC WALKING ON THE DOWNHILL A SLOPE

#### A. Outline of procedure

In a last work [14] and about the study of a kneeless biped robot with torso, we demonstrate that such systems can steer an almost passive dynamic walking on inclined slopes. Thus, the basic idea in this work is to do some transformation for the 7-dof biped robot. The main objective is to obtain nearly passive limit cycles, which will be exploited further to get active dynamic walking behaviour.
At heel strike, the impact is plastic, some energy is dissipated and support is transferred instantaneously. Because the model has a torso then an impulsive torque must be applied against the post-transfer stance leg to hold the torso in a desired region \[1\] \[2\]. Then we decide to examine the possibility that a kneed biped robot with torso can exhibit a passive dynamic walking in a stable gait cycle, downhill a slope, when torque is applied to stabilize the torso by a nonlinear feedback given in the next section (after locked the knees of both legs).

**B. Non collocated input/output linearization (partial)**

In this section we use some results from \[9\] \[10\] \[11\], the objective is to get a control scheme able to lock both knees and to stabilize the torso at a desired position.

To show this we may write the dynamic equations system (2) as:

\[
\begin{align*}
D_{11}\ddot{\theta}_1 + D_{12}\ddot{\theta}_2 + h_1 + \phi_1 &= 0 \\
D_{21}\ddot{\theta}_1 + D_{22}\ddot{\theta}_2 + h_2 + \phi_2 &= u
\end{align*}
\]

Where \(\theta_1 = q_1\) and \(\theta_2 = (q_{31}, q_{32}, q_{41}, q_{42})^T\) and:

\[
M(q) = \begin{bmatrix} D_{11} & D_{12} \\ D_{21} & D_{22} \end{bmatrix}
\]

is the symmetric, positive definite inertia matrix, the vector functions \(h_1(q,q)\in \mathbb{R}^1\) and \(h_2(q,q)\in \mathbb{R}^2\) contain coriolis and centrifugal terms, the vector functions \(\phi_1(q)\in \mathbb{R}^1\) and \(\phi_2(q)\in \mathbb{R}^2\) contain gravitational terms and \(u\) represents the input generalized force produced by the actuators.

From (9)-(10) we obtain:

\[
\begin{cases}
D_{11}\ddot{\theta}_1 + h_1 + \phi_1 = -D_{12}\ddot{\theta}_2 = -D_{121}\ddot{\theta}_{21} - D_{122}\ddot{\theta}_{22} \\
\ddot{\theta}_{21} = (\ddot{\theta}_{21}, \ddot{\theta}_{22}) = (v_{21}, v_{22})
\end{cases}
\]

Where

\[
\theta_{21} = (q_{31}, q_{32})^T \\
\theta_{22} = (q_{41}, q_{42})^T
\]

We verify that \(\text{rank}(D_{121}(q)) = 1\) for all \(q \in \mathbb{R}^5\), the system (9) is said to be strongly inertially coupled \[9\] \[10\] \[11\]. Under this assumption we may compute a pseudo-inverse matrix \(D_{121}^+ = D_{121}^T(D_{121}D_{121}^T)^{-1}\) and define \(v_{21}\) in (9):

\[
v_{21} = -D_{121}^+(D_{11}\ddot{\theta}_1 + h_1 + \phi_1 + D_{122}v_{22})
\]

With this choice of \(v_{21}\) we can write the following system:

\[
\begin{align*}
\ddot{\theta}_1 &= v_1, \quad \ddot{\theta}_2 = v_2 \\
D_{21}v_1 + D_{221}v_{21} + D_{222}v_{22} + h_2 + \phi_2 &= u
\end{align*}
\]

Thus we see that the passive degree of freedom \(q_1\) have been linearized and decoupled from the rest of the system. The actual control \(u\) is given by combining (9) and (10), after some algebra as:

\[
u = \tilde{D}_{21}v_1 + \tilde{D}_{222}v_{22} + \tilde{h}_2 + \tilde{\phi}_2
\]

\(\tilde{M}_{21}, \tilde{h}_2\) and \(\tilde{\phi}_2\) are given in Appendix B.

If \(q_1^d\) now represents a desired position of the torso, and \((q_{41}^d, q_{42}^d)\) represents the desired positions of both knees we may choose the additional controls \(v_1\) and \(v_{22}\) as:

\[
\begin{align*}
v_1 &= \dot{q}_1^d + k_d (q_1^d - q_1) + k_p (q_1^d - q_1) \\
v_{22} &= \dot{\theta}_{22}^d + k_d (\dot{\theta}_{22}^d - \dot{\theta}_{22}) + k_p (\theta_{22}^d - \theta_{22})
\end{align*}
\]

**C. Finding period one gait cycles and step period**

After locking both knees and stabilizing the torso we proceed to simulate the motion of the biped robot. The walker’s motion can exhibit periodic behaviour. Nearly passive Limit cycles are often created in this way (downhill a slope). At the start of each step we need to specify initial conditions \((q, \dot{q})\) such that after \(T\) seconds (\(T\) is the minimum period of the limit cycle) the system returns to the same initial conditions at the start. A general procedure to study the biped robot model is based on interpreting a step as a Poincaré map. Limit cycles are fixed points of this function.

A Poincaré map samples the flow \(\phi_\tau\) of a periodic system once every period \[20\]. The concept is illustrated in figure (2). The limit cycle \(\Gamma\) is stable if oscillations approach the limit cycle over time. The samples provided by the corresponding Poincaré map approach a fixed point \(x^*\). A non stable limit cycle results in divergent oscillations, for such a case the samples of the Poincaré map diverge.

\[
\text{Figure 2 Poincaré map}
\]

Let
\[ P = \Sigma \to \Sigma \]
\[ P(x_k) = x_{k+1} = \phi_z(x_k, T) \]  \hspace{1cm} (14)

Where the Poincaré hyperplane is defined by:
\[ \Sigma = \{(q, \dot{q}) \in \mathfrak{C} / x_1 = z_1, t > 0, q_1 = q_1^d, q_4 = q_4^d, q_2 = q_2^d \} \]

The fixed point \( x^* \) (initial conditions) can be located by the use of shooting methods \[20\].

D. Gait cycle stability

Stability of the Poincaré map (20) is determined by linearizing \( P \) around the fixed point \( x^* \), leading a discrete evolution equation:
\[ \Delta x_{k+1} = DP(x^*) \Delta x_k \]  \hspace{1cm} (15)

The major issue is how to obtain \( DP(x) \) - The Jacobean matrix- while the biped dynamics is rather complicated; a closed form solution for the linearized map is difficult to obtain. But one can be obtained by the use of a recent generalization of trajectory sensitivity analysis \[19\] \[20\] \[14\].

E. Numerical procedure

A numerical procedure \[19\] \[20\] \[14\] is used to test the walking cycle via the Poincaré map, it’s resumed as follows:
1. With an initial guess we use the multidimensional Newton-Raphson method to determine the fixed point \( x^* \) of \( P^* \) (immediately prior the switching event).
2. Based on this choice of \( x^* \), we evaluate the eigenvalues of the Poincaré map after one period by the use of the trajectory sensitivity.

F. Simulation results

Consider the model (1), we choose the hyper plane \( \Sigma \) as the event plane. We let the biped robot on a downhill slope with the control scheme (16). Starting with a suitable initial guess we obtain the following results:

Figure 4 Nearly passive limit cycle of the center of mass (Z) of the 7-dof biped robot

We took \( \gamma = 3^\circ, q_1^d = \frac{\pi}{7} \)
\[ K_p = 275, K_n = 30, K_n = 145, K_n = 25 \]
we obtain
\[ x^* = [-2.63 -3.25 -0.213 -0.213 -0.9 0 0.002 0 0.01 0.0012] \]

IV “CLC” FOR ACTIVE DYNAMIC WALKING ON THE LEVEL GROUND

We saw the existence of Nearly-passive limit cycles in section III. However these may not exist on all slopes, so some additional control is required. In this section we present “CLC” (Controlled Limit Cycles) \[1\] \[2\] \[3\], which considers the system energy for both controller design and system stabilization.

A. “CLC” Control

In order to get a periodic walk of the biped robot on the level ground we will use an additional control which will drive the zero dynamic to a reference trajectory characterized by the energy obtained from the Nearly-passive limit cycles.

We define a Lyapunov function as follows:
\[ V = \frac{1}{2} (E - E_{\text{ref}})^2 \] (16)

Where \( E \) is the total energy of the system defined as follows:

\[ E = \frac{1}{2} \dot{q}^T M(q) \dot{q} + \int_0^t \dot{q}^T \Gamma(q) \, dt \] (17)

and \( E_{\text{ref}} \) is the energy which characterizes the Almost-passive limit cycle.

\[ \dot{V} = (E - E_{\text{ref}})(E - E_{\text{ref}}) \] (18)

Where

\[ \dot{E} = \dot{q}^T Bu \]
\[ \dot{E}_{\text{ref}} = \dot{q}^T Bu_{\text{ref}} \] (19)

With

\[ u = D_{21} v_1 + D_{22} v_2 + \vec{h}_1 + \vec{\phi}_2 + \bar{u} \]
\[ u_{\text{ref}} = D_{21} v_1 + D_{22} v_2 + \vec{h}_1 + \vec{\phi}_2 \] (20)

\( \bar{u} \) in (27) represents the additional control. We choose the nonlinear control law:

\[ \bar{u} = -\Gamma \text{sign}(q^T B(E - E_{\text{ref}})) \] (21)

Where \( \Gamma > 0 \). It can be shown that the manifold defined by \( \{ \dot{\theta}_1 = \dot{\theta}_2 = 0 \} \cup \{ E = E_{\text{ref}} \} \) is attractive for the closed loop system and all trajectories converge towards the Almost-passive limit cycle which is a stable cycle exhibiting a periodic walk motion of the biped robot.

B. Simulation results

The control law (27) is applied to the system (7) on the level ground (\( \gamma = 0 \)). The next figures show that the CLC control leads to a stable walking motion of the kneed biped robot with torso (7-dof).

![Figure 4 The active limit cycle of the center of mass (Z) of the 7-dof biped robot](image)

V CONCLUSION

In this paper we present a control law, for a 7-dof biped robot, which realizes a stable continuous walking on the level ground to imitate a controlled nearly passive walking on the downhill slope. The last one is obtained by the use of a simple nonlinear feedback control with a numerical optimization. Next, the methodology developed uses Controlled Limit Cycles for stabilization of a periodic walk. The gaits correspond to limit cycles or region of attraction defined with the energy of the almost passive walking on the downhill slope. The approach is applied to the kneed biped robot with torso. Simulation results emphasize performance and efficiency of the proposed methodology. Future research intends to implement the same control law on a test bed biped robot.

VI REFERENCES


