Electromagnetic Forces in a Permanent Magnet Synchronous Machine with an Eccentric Rotor

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Abstract - Electric machinery comes in many different types and a strikingly broad range of sizes, from those little machines that cause cell telephones and pagers to vibrate to turbine generators with ratings upwards of a Gigawatt. In this paper we are working with a permanents magnets synchronous motor that is used in the traction of an electrical bicycle. Our purpose is to analyze the level and the shape of electromagnetic forces that appears inside the machine when it is in a good functioning estate and when the rotor is faulted.

I. Introduction

The electromagnetic forces acting between the rotor and stator when the rotor is eccentric comparing with the stator have been the point of interest and mathematical models have been developed almost through the whole history of the electrical machines. The mathematical models can be applied in analytical calculations or numerical analysis.

The electromagnetic forces witch appear in a permanent magnet synchronous motor are:

- The magnetostrictive force: - the materials utilised in the rotating machines are magnetostrictive only a little so we can neglect these forces.

- Maxwell’s force:

\[ dF = \frac{1}{2} \frac{d^2 A}{\mu_0} \]  

II. Analysis

The eccentric motions of the rotor, including stationary rotor displacement, can be described by whirling motions of the rotor. The cylindrical circular whirling motion means that the centerline of the rotor travels around the geometrical centerline of the stator in a circular orbit, with a certain frequency, known as the whirling frequency, and with a certain radius, known as the whirling radius [8]. The often mentioned basic modes, static
and dynamic eccentricity, are two special cases of whirling motion. The static eccentricity means that the rotor displacement is stationary comparing with the stator, and the whirling frequency is equal with zero. In dynamic eccentricity, the position of the minimum air gap rotates with the rotor, and the whirling frequency is the same as the mechanical angular speed [7] [8].

Knowledge of the electromagnetic forces at different whirling frequencies provides a good basis for designers and researchers with which to deal with some of the demands that exist for the electrical machines of the next generation.

The calculation of the electromagnetic forces has been a very popular research topic during the last decades [5]. The laws of electromagnetism are based on the Maxwell’s equations which make a link between the magnetic field $\mathbf{H}$ and the source of the current with the current density $\mathbf{J}$ and also between the electromagnetic induction $\mathbf{B}$ and magnetic vector potential $\mathbf{A}$:

\[
\begin{align*}
\mathbf{B} &= \text{rot} \mathbf{A} \\
\mathbf{J} &= \text{rot} \mathbf{H} \\
\text{div} \mathbf{B} &= 0 \\
\mathbf{B} &= \mu_0 \mu_r \mathbf{H}
\end{align*}
\]

(4)

There are two basic methods used to calculate the forces acting between the rotor and stator, namely methods based on the principle of the virtual work and methods based on the Maxwell’s stress [10]. For the first one we presume that the structure has an infinite length so for the induction vector we have only two components $B_x$ and $B_y$ similar with the field $\mathbf{H}$; presuming this we reduced the vectors $\mathbf{J}$ and $\mathbf{A}$ at the component which follows z direction [4] [5].

Combining the equations we have [1]:

\[
\begin{align*}
B_x &= \frac{\partial A}{\partial y} \\
B_y &= -\frac{\partial A}{\partial x} \\
(\text{rot} \mathbf{H})_z &= \frac{\partial H_x}{\partial x} - \frac{\partial H_y}{\partial y}
\end{align*}
\]

(5)

which conduces us at the Poisson’s differential equation:

\[
\frac{\partial}{\partial x} \left( \frac{1}{\mu_r} \frac{\partial A}{\partial x} \right) + \frac{\partial}{\partial y} \left( \frac{1}{\mu_r} \frac{\partial A}{\partial y} \right) + \mu_0 J = 0
\]

(6)

Euler’s theorem proves that the resolution of this equation consists of minimizing the energy function $W$, which is given by [10]:

\[
W = \int \int_{\Omega} \left[ \frac{1}{2 \mu_r} \left( \frac{\partial A}{\partial x} \right)^2 + \frac{1}{2 \mu_r} \left( \frac{\partial A}{\partial y} \right)^2 \right] - \mu_0 J A dxdy
\]

(7)

The integral is referred to the entire domain (all the surface) $\Omega$. We divide this surface on little elements using the finite element method (triangulation) [1] [6]. The potential in every point of one such element $(e)$ is defined such as the potential of three sums, depending of the coordinates of the triangle and the corresponding potentials, such as:

\[
A(e) = N_1 A_1 + N_2 A_2 + N_3 A_3 = \sum_{i=1}^{3} N_i A_i
\]

(8)

Minimizing energy function is possible when the partial derivatives concerning the potentials of every node are equal with zero:

\[
\frac{\partial W}{\partial A_i} = 0
\]

(9)

Equation (8) proves that we have a double integral on the domain which consists of $n$ elements $(e)$:

\[
\frac{\partial W}{\partial A_i} = \sum_{n} \int \left( \frac{1}{\mu_r} \frac{\partial A(e)}{\partial x} \frac{\partial A(e)}{\partial A_i} + \frac{\partial A(e)}{\partial y} \frac{\partial A(e)}{\partial A_i} \right) + \mu_0 J \frac{\partial A(e)}{\partial A_i} dxdy
\]

(10)
If we consider that \( J \) is uniform then the force is equal at [10]:

\[
F_i = \sum_n \int_{(c_i)} \mu_0 J N_i dxdy
\]

where \( c_i \) is the path of integration. In the calculation of forces and torques in the finite element analysis of electric devices, the methods based on the Maxwell stress tensor are commonly used. The electromagnetic force is obtained as a surface integral [8]:

\[
F = \int_{S} \sigma \cdot dS
\]

for a two-dimensional model, the surface integral is reduced to a line integral along the air gap [7] [8]. If a circle of radius \( r_i \) is taken as the integration path, the force is obtained from the equation:

\[
F = \int_{r_i}^{2r_i} \left[ \frac{1}{\mu_0} B_r B_\phi e_r + \frac{1}{2\mu_0} \left( B_r^2 - B_\phi^2 \right) e_\phi \right] rd\phi
\]

where \( B_r, B_\phi \) are the radial and tangential components of the flux density. If the solution were exact, the calculated force depends greatly on the choice of the integration radius. However, the force would be independent on the integration radius \( r_i \) when \( r_i \) varies within the air gap. The most approximates results are obtained if the line integral in equation (14) is transformed to a surface integral over the cross section of the air gap:

\[
F = \frac{1}{r_s - r_i} \int_{S_{ag}} \left[ \frac{1}{\mu_0} B_r B_\phi e_r + \frac{1}{2\mu_0} \left( B_r^2 - B_\phi^2 \right) e_r \right] dS
\]

where \( r_s \) and \( r_i \) are the outer and inner radii of the air gap and \( S_{ag} \) is the cross sectional area of the air gap. The drawback of the above method is the assumption of the rotational symmetry [8].

The motor studied is a permanent magnet synchronous machine with an exterior rotor [2] [9]. The structure and the characteristics of this motor are presented in Table II.1 and in Figure II.1:

<table>
<thead>
<tr>
<th>The characteristics</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Rated power (W)</td>
<td>200</td>
</tr>
<tr>
<td>Maximum speed (rot/min)</td>
<td>200</td>
</tr>
<tr>
<td>Rated load torque (N.m)</td>
<td>9.55</td>
</tr>
<tr>
<td>Maximum torque (N.m)</td>
<td>25</td>
</tr>
<tr>
<td>Rated current (A)</td>
<td>8.3</td>
</tr>
<tr>
<td>Efficiency (%)</td>
<td>70</td>
</tr>
<tr>
<td>Total mass of active parts (kg)</td>
<td>5.1</td>
</tr>
<tr>
<td>The mass of the magnets (kg)</td>
<td>0.9</td>
</tr>
<tr>
<td>Exterior diameter (m)</td>
<td>0.15</td>
</tr>
<tr>
<td>The length of the motor (m)</td>
<td>0.054</td>
</tr>
<tr>
<td>Number of poles pairs</td>
<td>6</td>
</tr>
<tr>
<td>Number of the slots</td>
<td>36</td>
</tr>
<tr>
<td>The induction in air gap (T)</td>
<td>0.69</td>
</tr>
</tbody>
</table>

For calculating and expressing the forces we are using FLUX2D programming environment, wich is using finite element theory [9]. The 200 W permanent magnet synchronous motor that we are using is demanding for a very complex network of triangulation so, at the end, we have approximately 43580 nodes and 21640 triangles.

**III. Results**

To observe the behavior in good condition estate of the permanent magnet synchronous machine we analyze the electromagnetic forces that appear inside the
motor. We obtained a regulate force; the regularity can emphasize the good behavior of the machine (Figure III.1):

![Figure III.1.](image)

The values of the forces acting in the magnets are included in the 644-702 N interval. The medium value of these forces is: $F = 670$ N.

Our purpose is to develop some parameters and after that to use these parameters for calculating the reliability and availability of the motor.

To follow our goals we faulted the machine to observe the electromagnetic forces acting inside the motor. We created four positions of the eccentric rotor and we decaled the rotor closer to the stator. At the beginning (in good functioning estate) we had all around an air-gap of 1 mm. we moved the rotor so that we can have a position where there is a smaller air-gap. In that place we decreased the air-gap with 0.1 mm; 0.15 mm; 0.2 mm and 0.25 mm (Figure III.2):

![Figure III.2.](image)

The unbalanced rotor creates a higher concentration of forces in the position mentioned above. Now we can calculate the magnetic pressure in the magnets. We observe that the magnetic pressure in the magnets area has increased (Table III.1):

<table>
<thead>
<tr>
<th>Position of the rotor</th>
<th>Magnetic pressure [N/mm$^2$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Good functioning estate</td>
<td>0.313</td>
</tr>
<tr>
<td>Decaled with 0.1 mm</td>
<td>0.370</td>
</tr>
<tr>
<td>Decaled with 0.15 mm</td>
<td>0.372</td>
</tr>
<tr>
<td>Decaled with 0.2 mm</td>
<td>0.379</td>
</tr>
<tr>
<td>Decaled with 0.25 mm</td>
<td>0.386</td>
</tr>
</tbody>
</table>

The fact that the magnetic pressure has increased leads us to observe the level of electromagnetic forces. Every step of the rotor closer to the stator increases these forces (Figure III.3).

The interval that includes these forces is also increasing. For the first movement of the rotor (Figure III.3.a) the values of the forces are included in the interval of 656 – 727 N, for the second movement (Figure III.3.b) we have an interval of 662 – 734 N, the third movement (Figure III.3.c) brings the level of the forces in the interval of 668 – 742 N and the last movement (Figure III.3.d) increases the forces up to 685 – 750 N. the forces that appear in the faulted machine are not so regular and there is a peak in the position where the rotor is closer to the stator; there the magnetic pressure increases and the electromagnetic forces are also increased.

In the Table III.2 we present the medium values of the forces and the distortion of these forces $\tau = \frac{F_{\text{max}} - F_{\text{min}}}{F_{\text{max}}}$ and also the level of the forces in percentage.

**IV. Conclusion**

For calculating the reliability and availability of an item we need certain parameters. The goal of this study is to analyze the level of the electromagnetic forces and of the magnetic pressure until the time when the machine is working in a proper state. We are able to observe that the forces
increase with every step of the rotor closer to the stator. In the future part of this study we will try to find a pattern for a multiple types of small electrical motors in which the level of the electromagnetic forces gives us the state of the machine.

![Graphs showing force vs time for different decals of the rotor](image)

**Table III.2.**

<table>
<thead>
<tr>
<th>Position of the rotor</th>
<th>Medium Forces values [N]</th>
<th>Level of the forces [%]</th>
<th>Distortion of the forces τ</th>
</tr>
</thead>
<tbody>
<tr>
<td>Good functioning estate</td>
<td>670 N</td>
<td>100 %</td>
<td>0.082</td>
</tr>
<tr>
<td>Decaled with 0.1 mm</td>
<td>682 N</td>
<td>102 %</td>
<td>0.097</td>
</tr>
<tr>
<td>Decaled with 0.15 mm</td>
<td>689 N</td>
<td>103 %</td>
<td>0.098</td>
</tr>
<tr>
<td>Decaled with 0.2 mm</td>
<td>695 N</td>
<td>104 %</td>
<td>0.099</td>
</tr>
<tr>
<td>Decaled with 0.25 mm</td>
<td>702 N</td>
<td>105 %</td>
<td>0.086</td>
</tr>
</tbody>
</table>

**V. Bibliography**

2. www.bin95.com
10. CEDRAT, “FLUX2D. Version 7.50. Tutoriaux de magnétostatique, de thermique permanent et de thermique évolutif d’électrostatique”.

**Figure III.3:**

a) rotor decaled 0.1 mm; b) rotor decaled 0.15 mm; c) rotor decaled 0.2 mm; d) rotor decaled 0.25 mm.