Measurement Information Recovery in Electrical Circuits and Systems Using Multiple Synchronous Detection Technique

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Abstract - *The paper develops the multiple synchronous detection technique used for measurement information recovery in electrical circuits and systems. The method consists in: generation of a complex excitation signal, with a known spectral structure, signal which, applied to the linear electrical system, will lead to a response-signal which will include the system's measurement information. This measurement information will be retrieve by multiple synchronous detection accordingly to the input signal's harmonics.*

The applications, of the method marked out in the paper, are: measuring of active and reactive components of a circuit, parasite components identification. The realizing technique could be in real time or displaced time.

1. Introduction

The behavior of an R, L, C circuit, seen between two points, is characterized by its equivalent circuit, in terms of impedance or admittance. The two components of the impedance have an either resistive (conductive) or reactive (susceptive) character. These components depend on frequency. The sinusoidal, alternating voltage and current, considered as equivalent quantities, associated with the resistive (conductive), respectively reactive (susceptive) elements, implicitly contain the measurement information into their amplitude and frequency.

A new generation of measuring systems for circuit parameters R, L, C does not rely on a traditional balancing technique. It is based on the principle of phase sensitive demodulation [1], [3]. Thus, the phase-vector projections of voltage and current, associated with the equivalent circuit (in terms of impedance or admittance), are determined in relation with a reference orthogonal phase-vector system. The signal processing is made by the agency of two synchronous detectors in quadrature.

A measuring instrument for circuit components based on the principle of synchronous detection requires only a resistive standard, as opposed with a bridge, which requires at least a resistive standard and a reactive one. On the other hand, in the case of the a.c. bridge, the relation between the orthogonal components of the standard impedance and the unknown impedance can not be accurately determined, because of the parasitic capacitances, which modify the real configuration of the bridge. In case of the synchronous detection in quadrature, this relation between the orthogonal circuit components is replaced by a relation between sinusoidal components of the quadrature signal. The accuracy of such a relation determination is achievable with low costs.

The synchronous detectors are achieved by chopping. They are controlled by two functions of synchronous commutation in quadrature. This operations can be achieved in real or displaced time, by memorising the input signals, aligned in time (implicitly with maintaining the phase shift), and then processing the signals on another time scale.

The commutation functions usually illustrated by square sinus functions delayed with $\pi/2$ can also be achieved in real or delayed time. The output signal is obtained by the agency of a low-pass filter with the cut-off frequency close to zero.

Recent publications [1], [2], [4] have studied the possible realization of an electrical system's complex response which could include completely the interest measurement information. Usually, this information is included into amplitude of certain harmonics of the response-signal. There are two retrieving methods of the measurement information: the synchronous detection method realized accordingly to interest harmonics (named multiple synchronous detection) and the spectral analyze method. Both methods lead to spectral amplitude's determination.

The first method offers the calibration advantage and, implicitly, the proof of the accuracy of measurement data. The spectral analyze method offers the advantage of using the Fast Fourier Transform and, implicitly, the use of a more compact implementations.

2. Principle of the method

Circuit parameters are defined starting from the associated electrical circuit models. These models can be extremely complicated, if all the parasite elements are considered: finite insulation resistance, capacitive or inductive reactance dependent on frequency. In most cases, simple models are used – series or parallel equivalent circuits. These simplifications, which neglect the parasite circuit elements, can be made by respecting some severe measurement conditions. For very low and very high impedances are used measuring circuit diagrams that highlight their definition as transfer impedances [3]. Figure 1 presents the two classical series and parallel models, for a circuit with two terminals.

The imaginary part of each equivalent circuit can be expressed in terms of capacitance or inductance. The following terms are also used: main component (L, C), respectively secondary component (R, G) for a circuit with reactive character and main component (R, G), respectively secondary component (L, C) for a circuit with resistive character. Another term that is used is quality factor Q, which is dependent on frequency. Another term also used is D factor, which is defined as a reciprocal value of the Q factor. For capacitances is used the tangent of the loss angle δ. Both Q and D are independent on the selected circuit model.

According to Figure 1, the following relations are known:

$$Z = R + jX, \quad Y = G + jB,$$
$$Z = \frac{1}{Y} = \frac{G - jB}{G^2 + B^2} = R + jX, \tag{1}$$
$$Y = \frac{1}{Z} = \frac{R - jX}{R^2 + X^2} = G + jB$$

Identifying the real and the imaginary parts, we obtain:

$$G = \frac{R}{R^2 + X^2}, \quad R = \frac{G}{G^2 + B^2},$$
$$B = \frac{X}{R^2 + X^2}, \quad X = \frac{B}{G^2 + B^2}, \tag{2}$$
$$Q = \frac{X}{R} = \frac{B}{G}, \quad D = \frac{1}{Q}$$

Meanwhile, the circuit parameters R, X, G, B include the information for loss resistances, parasite capacities and inductivities.

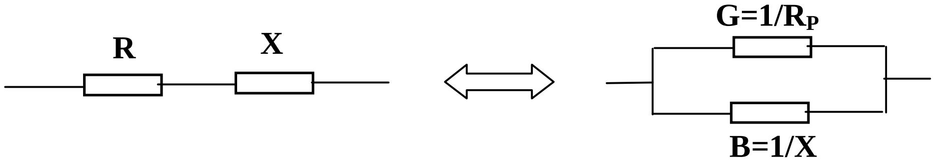

Fig. 1. Series and parallel equivalent circuits.

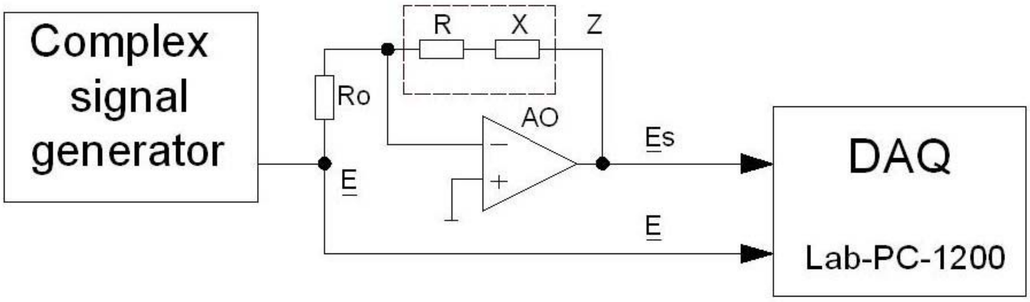

Fig. 2. Analogue primary measuring circuit for series configuration.

2.1. Primary measuring circuits

The alternating sine-wave voltage and current, associated with the impedance, respectively admittance of the circuit elements, and implicitly equivalent voltages and currents associated with resistive (conductive), respectively reactive (susceptive) elements contain the measuring information in amplitude and phase.

Primary measuring circuits for equivalent circuit configurations will be presented further on. These circuits have been chosen to utilise a single sinusoidal excitation source, the $\underline{E}$ voltage.

a) Series configuration

The primary measuring circuit for prominence the circuit's parameters of the equivalent series model is illustrated in Figure 2.

The signal generator generates a signal sum of harmonics of known amplitudes and having the following format:

$$e(t) = E_1 \sin(\omega_0 t + \varphi_1) + E_2 \sin(2\omega_0 t + \varphi_2) + \\ + ... + E_k \sin(k\omega_0 t + \varphi_k) \quad (3)$$

In complex, we have:

$$e(t) \leftrightarrow \underline{E} = \underline{E}_1 + \underline{E}_2 + ... + \underline{E}_k \quad (4)$$

In [1], [2], [4] empirical conditions for choosing the $\underline{E}_k$ amplitudes and the φ_k phase-difference are imposed, thus a resultant signal with a reduced crest factor is obtained. It is important to know and to control the effective value of the $e(t)$ tension.

From the Figure 2, we have:

$$\underline{E}_s = -\frac{\underline{E}}{R_0} Z = -[\frac{\underline{E}}{R_0} R + j\frac{\underline{E}}{R_0} X] \quad (5)$$

or

$$-\underline{E}_s = \underline{E}_R + j\underline{E}_R \frac{X}{R} = \underline{E}_R + j\underline{E}_X \quad (6)$$

Relation (5) marks out the fact that circuit parameters R and X are included into amplitudes of two quadrature tensions. It results that R and X could be determined by a quadrature synchronous detection method accordingly with a reference signal in phase with $\underline{E}$ signal. This fact is feasible into a displaced-time technique, by discreet-time processing.

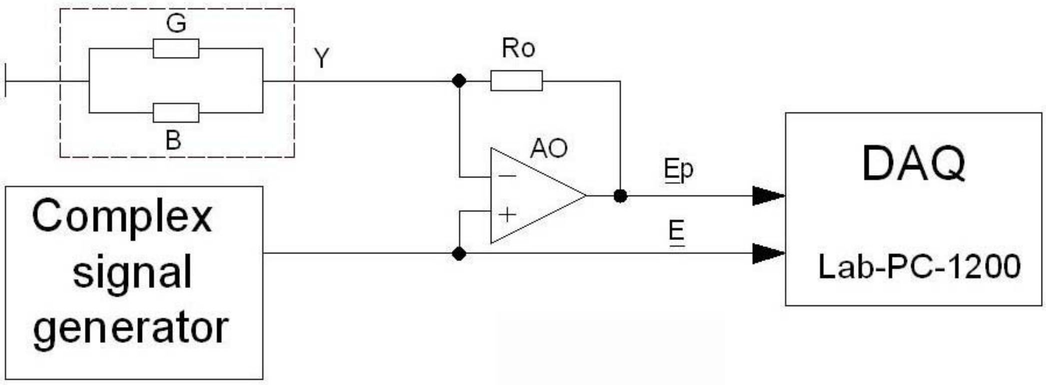

Fig. 3. Analogue primary measuring circuit for parallel configuration.

b) Parallel configuration

In Figure 3 is presented the primary measuring circuit for the parallel configuration of G and B parameters. The amplifier AO is in configuration of non-inverting amplifier.

The output voltage $\underline{E}_p$ has the expression:

$$\begin{aligned}\underline{E}_p &= \underline{E}(1+R_0Y) = \underline{E}[1+R_o(G+jB)] = \\ &= \underline{E}(1+R_0G) + j\underline{E}R_0B\end{aligned} \tag{7}$$

We also notice that voltage $\underline{E}_p$ has two harmonic components in quadrature, which contain the measurement information G and B.

2.2. Utilization of virtual technique in complex signals demodulation

The phase sensitive demodulation in virtual technique includes the following operations: sampling of the signal carrying information and of the reference signal, making up two numerical sequences associated with these samples, multiplication of the numerical sequences and finally, extraction of the numerical sequence of the demodulated signal throughout numerical filtering operations. Further on, one component of the complex signal will be considered.

Considering the signal carrying information expressed by:

$$v_x(t) = x(t)\cos(\omega t - \varphi) \tag{8}$$

where: x(t) is the amplitude; φ - the phase difference reported to the reference signal phase; ω - angular frequency (ω = 2πf, f – frequency).

Developing the relation (8), we obtain:

$$v_x(t) = x_c(t)\cos\omega_x t + x_s(t)\sin\omega_x t \tag{9}$$

where

$$x_s(t) = x(t)\sin\varphi, \quad x_c(t) = x(t)\cos\varphi$$

In complex we can write:

$$\underline{v_x}(t) = \underline{x_c}(t) + j\,\underline{x_s}(t) \tag{10}$$

We consider that the measurement information is contained in the amplitude x(t) and in the phase φ of the signal $v_x(t)$. This thing is equivalent with considering that the measurement information is included in the amplitudes of the orthogonal signals $x_c(t)$, respectively $x_s(t)$. The values of these amplitudes will be determined through the agency of two complex phase sensitive demodulators (synchronous detectors in quadrature).

In this paper, two types of synchronous detectors in virtual technique have been synthesised: synchronous detector achieved by direct multiplexing of the samples (DSD) and synchronous detector achieved by multiplexing through chopping (DSC).

2.2.1. Synchronous detector achieved by direct multiplexing of the samples – DSD

The algorithm of synthesis in virtual technique of the synchronous detector through multiplexing of the samples is made starting from the synthesis of the analogue phase sensitive demodulator, achieved by analogical multiplexing of signals. We consider:

- input analogue signal $v_x(t)$ given by the relation (9)
- reference signal, phase origin $x_{r1}=X_r \cos(\omega t)$
- reference signal in quadrature $x_{r2}=X_r \cos[\omega(t-T/4)]$, where T is the period of signals $v_x(t)$ and $x_{r1}(t)$, $T = 2\pi/\omega$.

Be the sampling rate equal with n_0 (samples per second).The number of samples within a period is $N_0 = n_0T$. The sampling duration is $T_e = 1/n_0$.

In Figure 4 is presented the diagram of the synchronous detector in quadrature, achieved through direct multiplication of samples, where:

S-H – sample-hold block
ADC – analogue-digital converter
LPF – low-pass filter

Two channels for drawing the signal v_x and the reference signal x_{r1} are used. The samples' drawing is made synchronously for the two channels. At the output of analogue-

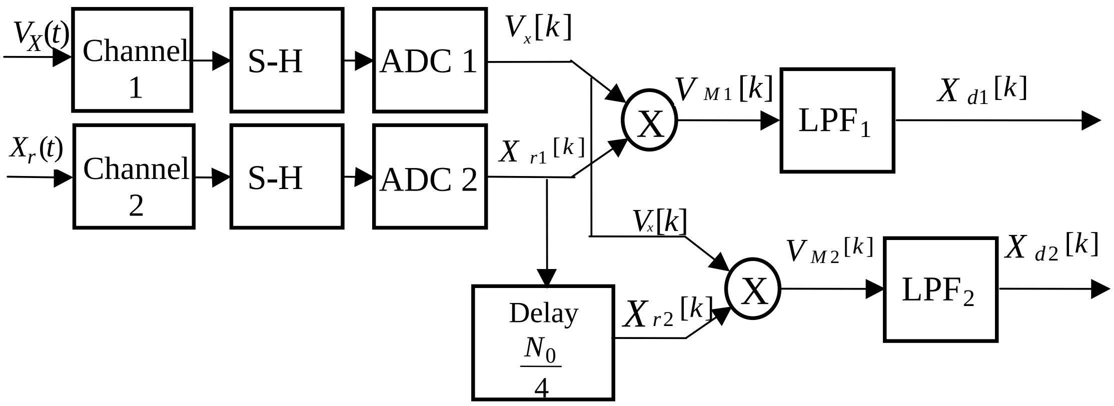

Fig. 4. Virtual technique quadrature synchronous detector, achieved by direct multiplication of samples.

digital converter the numerical sequences are obtained:

$$V_x[k]=\{V_x(k)\},\quad x_{r1}[k]=\{x_r(k)\},\qquad k=\overline{1,N} \tag{11}$$

where N is the total amount of drawn samples; $v_x(k)$ and $x_r(k)$ are samples of order k of input and reference signals.

The numerical sequence of reference signal in quadrature is obtained through an operation of back displacement with $N_0/4$ samples of the signal sequence $x_{r1}[k]$:

$$x_{r2}[k]=x_{r1}\left[k-\frac{N_0}{4}\right],\quad k=\overline{1,N} \tag{12}$$

After the multiplication of the sequences $v_x[k]$, $x_{r1}[k]$, respectively $v_x[k]$, $x_{r2}[k]$, the sequences $v_{M1}[k]$, $v_{M2}[k]$ are obtained.

$$\begin{aligned} V_{M1}[k]&=\{V_x[k]\cdot V_{r1}[k]\},\\ V_{M2}[k]&=\{V_x[k]\cdot V_{r2}[k]\},\quad k=\overline{1,N} \end{aligned} \tag{13}$$

These numerical sequences represent input signals in numerical filters LPF$_1$, LPF$_2$. At the output of the low-pass filter, the couple of sequences $x_{d1}[k]$, respectively $x_{d2}[k]$ is obtained, which represents the complex signal numerical demodulated.

The reconstruction of the measurement information contained in the amplitude x(t) and the phase φ is achieved by determination of samples x(k) and φ(k).

$$\begin{aligned} x(k)&=\frac{2}{X_r}\sqrt{x_{d_1}^2(k)+x_{d_2}^2(k)}\\ tg\varphi(k)&=\frac{x_{d_2}(k)}{x_{d_1}(k)} \end{aligned} \tag{14}$$

Finally, the sequences x[k], respectively φ[k] are:

$$x[k]=\{x(k)\},\quad \varphi[k]=\{\varphi(k)\},\quad k=\overline{1,N} \tag{15}$$

Further on we obtain:

$$\begin{aligned} x_s(k)&=x(k)\cdot\sin\varphi(k),\\ x_c(k)&=x(k)\cdot\cos\varphi(k),\\ x_s[k]&=\{x_s(k)\},\quad x_c[k]=\{x_c(k)\}, \end{aligned} \tag{16}$$

Observation: If the amplitude of the input signal x(t)=const., the samples $x_{d1}(k)$, $x_{d2}(k)$ are also constant and they can be obtained through an arithmetic averaging operation applied to the elements of the sequences $v_{M1}[k]$, respectively $v_{M2}[k]$. Implicitly, the filtration numerical operators LPF$_1$, LPF$_2$ will be replaced by averaging algebraic operators. In this case, the number of samples should be increased in order to diminish the discrete representation error. Experimentally, this can be simply achieved by increasing the sampling rate.

2.2.2. Chopping synchronous detector in virtual technique

In this case too, the starting point is the analogue phase sensitive demodulator, achieved through chopping. It is known that the multiplication through chopping consists in the multiplication of input signal with 1, respectively with –1, on the duration of the first half of the period, respectively the second half of the period of the reference signal [7]. This is equivalent with the multiplication of the input signal with the

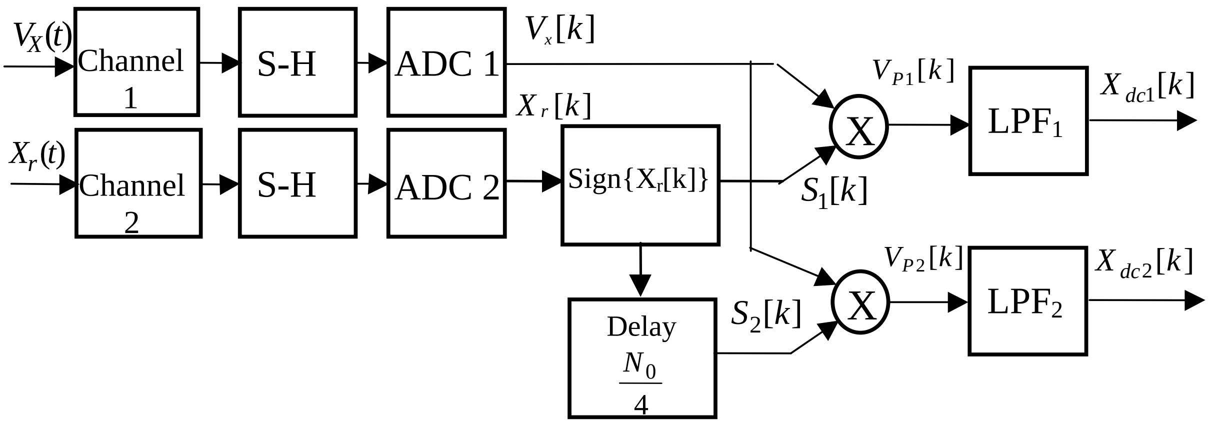

Fig. 5. Virtual technique quadrature chopping synchronous detector.

signal resulted from the application of "signum" function to the reference signal. In Figure 5 is illustrated the circuit diagram of synchronous detector in quadrature in virtual technique, achieved through chopping.

In comparison with the synchronous detector illustrated in Figure 4, we notice the constitution of the sequences $s_1[k]$, $s_2[k]$. A sample of the sequence $s_1[k]$ is obtained thorough the virtual operator

$$S_1[k] = \text{sign } X_r[k] \tag{17}$$

The sample of k order, $s_2[k]$, is obtained through an operation of delay with $N_0/4$ samples of the sample $s_1[k]$:

$$S_2[k] = S_1[k - N_0 /4] \tag{18}$$

Finally, the sequences $s_1[k]$, $s_2[k]$ are constituted through the making up of some sampling successions:

$$S_1[k] = \{S_1(k)\}, \quad S_2[k] = \{S_2(k)\} \tag{19}$$

The operation of virtual chopping consists in the multiplication of the sequences $v_x[k]$, $s_1[k]$, respectively $v_x[k]$, $s_2[k]$. The sequences $v_{p1}[k]$, respectively $v_{p2}[k]$ are obtained:

$$\begin{aligned} V_{P1}[k] &= \{V_x(k)\cdot S_1(k)\}, \\ V_{P2}[k] &= \{V_x(k)\cdot S_2(k)\}, \quad k = \overline{1,N} \end{aligned} \tag{20}$$

The signal in quadrature, numerically demodulated, is expressed by the pair of numerical sequences $x_{dc1}[k]$, respectively $x_{dc2}[k]$. These sequences are obtained at the output of the numerical filters LPF$_1$, LPF$_2$.

We notice that between the sequences $x_d[k]$ and $x_{dc}[k]$ there is the following relation:

$$x_d[k] = \frac{4}{\pi} x_{dc}[k] \tag{21}$$

3. Examples

The method will be illustrated by two examples.

a) Measurement of R, L, C of a series circuit

Let us consider the complex signal of two components $\underline{E}_1$ and $\underline{E}_3$ of frequencies ω_0 and $3\omega_0$. Let us consider both amplitudes equals and both initial phases equal to zero, i.e.:

$$\underline{E} = \underline{E}_1 + \underline{E}_3 \leftrightarrow e(t) = E\sin\omega_0 t + E\sin 3\omega_0 t, \quad \omega_0 = 2\pi f_0 \tag{22}$$

The primary measuring circuit is the one illustrated in Figure 2.

The series circuit's impedances at ω_0 and $3\omega_0$ frequencies are:

$$Z_1 = R + j(\omega_0 L - \frac{1}{\omega_0 C}) = R + jX_1 \tag{23}$$

$$Z_3 = R + j(3\omega_0 L - \frac{1}{3\omega_0 C}) = R + jX_3 \tag{24}$$

It infers:

$$C=\frac{8}{3}\frac{1}{\omega_0(X_3-3X_1)} \quad (25)$$

$$L=\frac{3}{8}\frac{1}{\omega_0}(X_3-\frac{X_1}{3}) \quad (26)$$

Returning to the Figure 2 circuit, it observes that for $\underline{E}$ given by relation (22), the circuit's equation (relation (5)) could be written like a sum of two tensions:

$$\underline{E}_S=\underline{E}_{S1}+\underline{E}_{S3} \quad (27)$$

where

$$-\underline{E}_{S1}=\underline{E}_1\frac{R}{R_0}+j\underline{E}_1\frac{X_1}{R_0}=\underline{E}_{R1}+j\underline{E}_{X1} \quad (28)$$

$$-\underline{E}_{S3}=\underline{E}_3\frac{R}{R_0}+j\underline{E}_3\frac{X_3}{R_0}=\underline{E}_{R3}+j\underline{E}_{X3} \quad (29)$$

Measuring X_1 and X_3 components becomes to measuring $\underline{E}_{X1}$ and $\underline{E}_{X3}$ tensions. Resistance R is indirectly determined by measuring one of $\underline{E}_{R1}$ or $\underline{E}_{R2}$. It results a measuring scheme using three synchronous detectors, as shown in Figure 6.

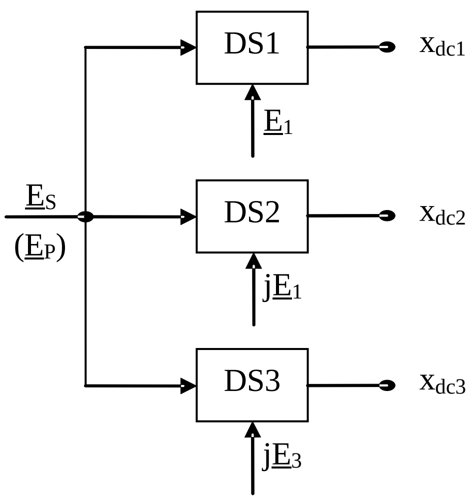

Fig. 6. Measurement block-scheme for R, X_1, X_3 (G, B_1, B_3) components.

Let us consider the synchronous detectors realized by chopping (as shown in Figure 4). The equations of the three synchronous detectors are:

$$x_{dc1}=\frac{2}{\pi}E_{R1}=\frac{2}{\pi}E_1\frac{R}{R_0} \quad (30)$$

$$x_{dc2}=\frac{2}{\pi}E_{X1}=\frac{2}{\pi}E_1\frac{X_1}{R_0} \quad (31)$$

$$x_{dc3}=\frac{2}{\pi}E_{X3}=\frac{2}{\pi}E_3\frac{X_3}{R_0} \quad (32)$$

Considering $E_1=E_3$, it infers:

$$R=\frac{\pi}{2}\frac{R_0}{E}x_{dc1} \quad (33)$$

$$X_1=\frac{\pi}{2}\frac{R_0}{E}x_{dc2} \quad (34)$$

$$X_3=\frac{\pi}{2}\frac{R_0}{E}x_{dc3} \quad (35)$$

These results, introduced in (24), (25), (26), lead to determination of L and C values.

b) Measurement of G, L, C of a parallel circuit

The primary measurement circuit is illustrated in Figure 3. The admittance of the circuit activated by the same signal (22) has two components:

$$Y_1=G+j(\omega_0C-\frac{1}{\omega_0L})=G+jB_1 \quad (36)$$

$$Y_3=G+j(3\omega_0C-\frac{1}{3\omega_0L})=G+jB_3 \quad (37)$$

It infers:

$$C=\frac{3}{8\omega_0}(B_3-\frac{B_1}{3}) \quad (38)$$

$$L=\frac{8}{3\omega_0}\frac{1}{(B_3-3B_1)} \quad (39)$$

The primary measurement circuit's equation – relation (7) – is written as a sum of two components $\underline{E}_{P1}$ and $\underline{E}_{P3}$:

$$\underline{E}_{P1}=\underline{E}_1(1+R_0G)+j\underline{E}_1R_0B_1 \quad (40)$$

$$\underline{E}_{P3}=\underline{E}_3(1+R_0G)+j\underline{E}_3R_0B_3 \quad (41)$$

Determining the B_1, B_3 values means to measure the quadrature components of $\underline{E}_{P1}$ and $\underline{E}_{P3}$ tensions. Determining the G value means to measure one of the components in phase of $\underline{E}_{P1}$ and $\underline{E}_{P3}$ tensions.

The measurement scheme is similar with the one from Figure 7, with the observation that the input tension is $\underline{E}_P$.

After simple calculations we obtain:

$$G = (\frac{\pi}{2}\frac{x_{dc1}}{E} - 1)\frac{1}{R_0} \quad (42)$$

$$B_1 = \frac{\pi}{2}\frac{x_{dc2}}{E}\frac{1}{R_0} \quad (43)$$

$$B_3 = \frac{\pi}{2}\frac{x_{dc3}}{E}\frac{1}{R_0} \quad (44)$$

These results, introduced in (38), (39) lead to the determinations of the L, C values for the equivalent parallel circuit.

4. Conclusions

The virtual technique for measuring circuit parameters through a synchronous detection method described in this paper represents an illustration of performances of the graphic programming medium LabVIEW (National Instruments). Certainly, the variant presented by the authors represents a possibility of achievement, which can mean in the same time a work project that can be continued, developed and improved.

The distinctions of this measuring device are: the easiness of the design, the flexibility of interconnecting the blocks for signals soft processing, the elegance of presenting the results on the soft panel of the virtual instrument and the accuracy of soft data processing. The measurement precision is determined by errors pluralism proceeded from the analogue measuring circuit, model errors and interaction errors. The errors proceeded from data processing are insignificant.

As a novelty, the paper highlights two possibilities for implementing the synchronous detectors in virtual technique. The relation between the active (R, G) and the reactive (X, B) elements of the circuit is reduced to a relation between two sinusoidal orthogonal signals. These signals include in their amplitude the measurement information – which is extracted through the phase sensitive demodulation operation.

In conclusion, the paper has the main merit of applying a new philosophy in designing the measurement instrumentation, namely the virtual technique for signal processing. We hope that our work joins other achievements using the LabVIEW programming technique.

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