An 8/6 Switched Reluctance Motor Calculations Using a Combined FE-BE Methods

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***Abstract**: The paper deals with a proposed coupled FE-BE method based on the coupling of the finite element and boundary element methods to compute parameters such as flux linkage, inductance and static torque of an 8/6 switched reluctance motor (SRM). The combination of both techniques in the same computer program is the most efficient way of tackling problems that contain features requiring both FE and BE capabilities. The conducting and magnetic parts of the motor are discritized by the finite element whereas the boundary element is used to discritize its air gap. The interfaces between the two methods' domains were chosen to be circular to avoid the discontinuity of the normal vector at the corners of poles. To illustrate the merits of such proposed coupled FE-BE method, the obtained simulation results are compared to the prototype design values of this motor as cited in a given reference.*

***Keywords**: Switched reluctance motor, finite element method, boundary element method, static torque.*

1. INTRODUCTION

Switched Reluctance Machines (SRMs) are receiving more attention from industries in recent years. They are less expensive, reliable and weigh less than other machines of comparable power outputs [11].

A reluctance motor is an electric motor in which torque is produced by the tendency of it moveable part to move to position where the inductance of the excited winding is maximized.

To investigate the SRM performance, including the design of the control circuit, it is necessary to know its parameters, such as flux linkage, inductance and static torque.

With classical methods, electrical machines are generally simulated using models based on lumped parameters in an electrical circuit. However, errors are inevitable when expressing field characteristics using electrical circuit models [6]. Numerical techniques are thus employed to avoid many assumptions in those circuit models. The tedious process of making and modifying the prototypes becomes unnecessary with the advent of numerical techniques. The precious time that is otherwise required to design, test, and market a quality product is greatly reduced when accurate numerical models are employed.

The finite and boundary element methods are therefore used as a very powerful tool to replace costly experimentally based design with simulation based design. The FEM is capable to handle structure having complexed geometries, nonlinear material and complicated boundary conditions. However it sometimes presents some difficulties in particular with the mesh generation in the air gap which is very small in comparison to the main dimension of the stator and rotor and also for rotating device where for each rotor position we have to remesh the entire domain. In spite of its limitation to handle nonlinearity, BEM possess several important advantages. Among these advantages, the need to discritize only the interfaces between different media and the space between moving objects does not need to be meshed. The combination of

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the two numerical techniques allows us to benefit further from the advantages of each of them. In this paper, we present a 2D numerical method coupling the isoparametric FEM and BEM of second degree for the calculation of electromagnetic field in an 8/6 switched reluctance motor. Our domain of study is divided in three main parts.

1. An outer cylinder containing the stator laminations. Both conductors and air in this part are discritized with the FEM.
2. An intermediary ring of air, discritized with the BEM.
3. An inner cylinder containing the rotor laminations. Both shaft and air are discritized with the FEM.

With this approach, the stator and rotor mesh are disconnected and are meshed separately. For our simulation work and for the thirteen positions of the rotor, the stator and rotor are only meshed once. The connectivity table is unchanged and only the coordinate tables of the rotor's nodes will be changed easily by multiplying each coordinate couple (x, y) by the rotation matrix $R(\theta)$

$$R(\theta) = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}$$

where θ is the angle of rotation of the rotor.

2. FORMULATION OF THE PROBLEM

To determine the magnetic field distribution of the SRM, the following assumptions are made: (1) The magnetic field distribution is constant along the longitudinal direction of the SRM; (2) the magnetic field outside the SRM periphery is negligible and has zero magnetic vector potential, as the Dirichlet boundary condition is assigned to the outer periphery of the SRM surface; (3) in the two-dimensional analysis, the current density vector has a component only in the z direction, and, for this reason, the flux density vector has components only in the *x* and y directions.

The field solution is obtained by computing the vector potential A in Cartesian coordinates (x, y) described by the $2-D$ Poisson's equation:

$$\frac{\partial}{\partial x}(\nu(B)\frac{\partial A}{\partial x}) + \frac{\partial}{\partial y}(\nu(B)\frac{\partial A}{\partial y}) = -J(x, y) \quad (1)$$

2.1. FEM discretization

The cylindrical stator and rotor domains are meshed using isoparametric second order triangular elements.

Starting from equation (1), the Galerkin-formulation yields, after integrating by parts and discretization [5].

$$\sum_{\Delta}(\sum_{k=1}^{r}\int_{\Omega_j}(\nu\nabla N_i.\nabla N_k)d\Omega\, A_k - \sum_{k=1}^{l}\int_{\Gamma_j}\nu N_i N_k d\Gamma\,(\frac{\partial A}{\partial n})_k) = \sum_{\Delta}\int_{\Omega_j} -JN_i d\Omega \quad (2)$$

i=1…r, r=number of element nodes, j=number of element, *l*=number of element nodes on Γ_j, Γ_j is the intersection of Ω_{BEM} with the element .The trial function used for A and $\frac{\partial A}{\partial n}$ are

$$A = \sum_{k=1}^{r} N_k A_k\ , \quad \frac{\partial A}{\partial n} = \sum_{k=1}^{l} N_k (\frac{\partial A}{\partial n})_k \quad (3)$$

Equation (2) written in matrix form is

$$[K]\{A_k^{FEM}\} - [T]\left\{\left(\frac{\partial A}{\partial n}\right)_k^{FEM}\right\} = \{F^{FEM}\} \quad (4)$$

In (4) there are *n* unknowns A_k^{FEM} and *m* unknowns $\left(\frac{\partial A}{\partial n}\right)_k^{FEM}$.

2.2. BEM discretization

The boundary element method is applied to the ring shaped air gap. Thus $\nu = \nu_0 = \frac{1}{\mu_0}$, $J = 0$ and Poisson's equation becomes Laplace's equation

$$\Delta A = 0 \quad (5)$$

The corresponding integral formulation for (5), after two integration by parts [5], [12], is

$$\int_{\Omega_{BEM}} A\Delta u^* d\Omega + \oint_{\Gamma_{BEM}} u^* \left(\frac{\partial A}{\partial n}\right) d\Gamma - \oint_{\Gamma_{BEM}} \left(\frac{\partial u^*}{\partial n}\right) A d\Gamma = 0 \quad (6)$$

Where

$$u^* = \frac{1}{2\pi}\ln\frac{1}{r} \quad (7)$$

u^* =weighting function (fundamental solution) and r is the distance from the source point to the field point.

After applying the weighting function u^*, putting the source point to the boundary and taking into account the singularities of the integrands, we get the discritized formulation

$$c_k A_k + \sum_{j=1}^{p} \int_{\Gamma_j} \frac{\partial u^*}{\partial n} A d\Gamma = \sum_{j=1}^{p} \int_{\Gamma_j} u^* \frac{\partial A}{\partial n} d\Gamma \quad (8)$$

p=number of elements, $\Gamma_{BEM} = \sum_{j=1}^{p} \Gamma_j$. The source will be put successively at every node, i.e. *k=1…m*, which yields

$$[H]\left\{A_k^{BEM}\right\} = [G]\left\{\left(\frac{\partial A}{\partial n}\right)_k^{BEM}\right\} \quad (9)$$

The order of matrices in (9) is $m \times m$.

Multiplying (9) by $[G]^{-1}$ results in

$$\left\{\left(\frac{\partial A}{\partial n}\right)_k^{BEM}\right\} = [G]^{-1}[H]\left\{A_k^{BEM}\right\} \quad (10)$$

In the common interfaces we have the boundary conditions

$$\left\{A_k^{FEM}\right\} = \left\{A_k^{BEM}\right\} \quad (11)$$

And

$$\left\{\nu_{FEM}\left(\frac{\partial A}{\partial n}\right)_k^{FEM}\right\} = -\left\{\nu_0\left(\frac{\partial A}{\partial n}\right)_k^{BEM}\right\} \quad (12)$$

The minus sign in (12) results from the orientation of the normal vector.

With the choice of compatible element for the interface between Ω_{FEM} and Ω_{BEM}, we eliminate the unknowns $\left(\frac{\partial A}{\partial n}\right)_k$ from (4), which is now read as

$$\left([K] + [T][G]^{-1}[H_\nu]\right)\left\{A_k\right\} = \left\{F^{FEM}\right\} \quad (13)$$

After imposing the Dirichlet boundary condition ($A = 0$) on (13), the resulting non-linear system is solved for the unknowns A_k by the Newton-Raphson method.

This coupled FE-BE method was applied to the calculation of the flux linkage, the inductance and static couple of the SRM as presented in Table I.

TABLE I. MAIN DIMENSION OF THE SRM.

Number of Stator Poles:	8
Number of Rotor Poles:	6
Stator pole arc	18 degrees
Rotor pole arc	22 degrees
Outer stator diameter	190 mm
Bore diameter	100.6 mm
Stator back iron thickness	12 mm
Height of stator pole	32.7 mm
Height of rotor pole	19.8
Air gap length	0.5 mm
Shaft diameter	28 mm
Stack length	200 mm
Power output	5hp
Conductor area of cross section	1.588 mm^2

3. PARAMETERS EVALUATION

3.1. Calculation of the flux and inductance

Let us write the magnetic field $\mathbf{B}$ in terms of a vector potential $\mathbf{A}$, so that

$$\mathbf{B} = \nabla \times \mathbf{A} \quad (14)$$

It follows from Stokes' theorem that the flux entering the stator pole is

$$\Phi = \int_{pole} \mathbf{B.ds} = \int_{pole} \nabla \times \mathbf{A.ds} = \oint_{pole} \mathbf{A.d}l \quad (15)$$

The flux per unit length in 2D models is given by

$$\Phi_l = A_1 - A_2 \quad (16)$$

Where 1 and 2 are the extreme lines of the flux tube entering the pole.

The flux linkage and the inductance are given by:

$$\lambda = N_c l_s \Phi_l \quad (17)$$

$$L = \frac{\lambda}{i} \quad (18)$$

N_c is the number of turns per phase, l_s is the stack length and I is the phase current.

3.2. Calculation of the static torque

In electrical engineering the calculation of forces and torques is a subject of great importance. Direct calculation of torque in SRM with analytical method is impossible. Usually, torque is derived from the flux linkage characteristic of phase [10]. However, analytical methods cannot model precisely the region in were the stator and rotor poles begin to overlap, so prediction of static torque is faced with a problem in that region.

With constant current and for a *2D* domain, the torque can be obtained by the derivation of the magnetic co-energy W_c by unity length l [8].

$$T = l\frac{dW_c}{d\theta_m} \tag{19}$$

l is the length perpendicular to the studied plane and θ_m the angle defining the relative position between rotor and stator.

Numerically, the derivation in (19) is approximated by the following equation

$$T = \frac{W_c\left(\theta_m\right) - W_c\left(\theta_m - \Delta\theta\right)}{\Delta\theta} \tag{20}$$

$$W_c = W_{FEM} + W_{BEM} \tag{21}$$

$$W_{FEM} = \int\limits_{\Omega_{FEM}} \left(\int B(H)dH \right) ds = \sum_{i=1}^{N} \left(\int B(H)dH \right) \Delta s_i + \frac{1}{2}\sum_{i=1}^{M}\left(\frac{B_i^{\,2}}{\mu_0}\right)\Delta s_i \tag{22}$$

$$W_{BEM} = \frac{1}{2}\sum_{j=1}^{L}\left(\frac{B_j^{\,2}}{\mu_0}\right)\Delta s_j \tag{23}$$

N is the number of finite elements in the nonlinear domain and M is the number of finite elements in air.

The boundary element domain is divided in L curvilinear rectangles.

4. NUMERICAL RESULTS

A computer program based on the above formulation is developed and used to compute the magnetic parameters of the SRM of Table I. In order to test the performance of this program, the calculated values are compared to values obtained by FLUX 2D used for the design verification of the SRM of table I and cited in references [11].

Since the poles number of the rotor is 6, the magnetic circuit becomes the same after rotating 60 degrees, when only one phase is excited. The stator salient pole and rotor salient pole repeat facing conditions and non facing condition for the interval of rotation 30 degrees. The fully aligned position of the stator pole and rotor pole is defined as 0 degree, and the rotor is made to rotate from 0 degree to 30 degrees. For each 2.5 degrees we compute the flux linkage, the inductance and the static torque for two current levels 10A and 16A.

4.1. Flux plot

A very interesting way to check he results of a magnetic simulation is to visualize the flux distribution. It is a powerful tool to check the validity of the numerical results. Flux lines

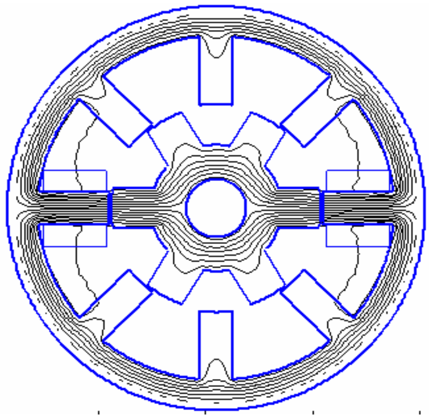

Fig. 1. Flux plot at fully aligned position with phase I excited.

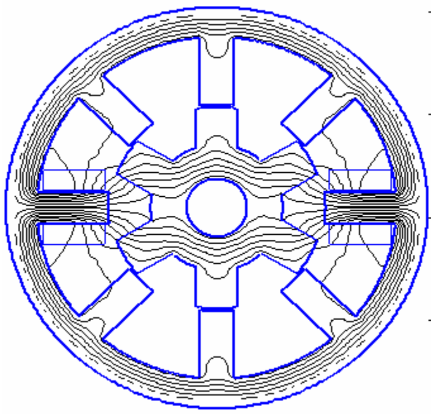

Fig. 2. Flux plot at fully unaligned position with phase I excited.

for two position of the rotor corresponding to the maximum and minimum values of the inductance are shown in Fig.1 and Fig.2.

4.2. Flux linkage and inductance

Figures 3 to 5, show respectively the flux linkage versus rotor position for two current levels, the flux linkage versus phase current for the facing and non facing positions and the inductance versus rotor position for the two current levels.

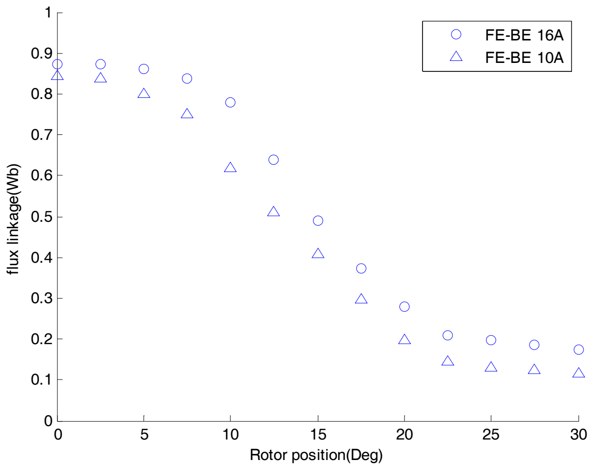

Fig. 3. Flux linkage versus rotor position.

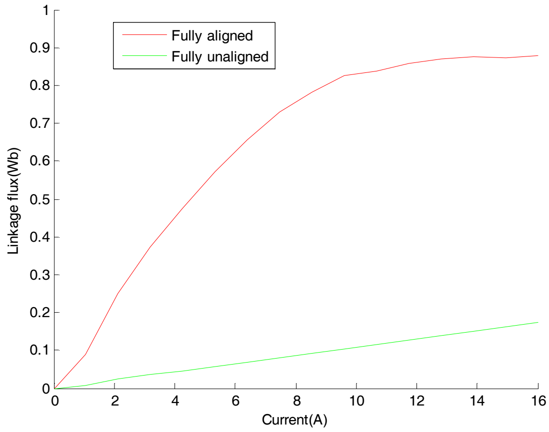

Fig. 4. Flux linkage versus current.

The waveform of Fig (5) show that the inductance does not change in the unaligned position where the air gap is very big and no iron saturation takes place in the magnetic circuit. But in the aligned position where the air gap in very narrow, the iron saturation causes the changing of the inductance value with current, hence the inductance is a

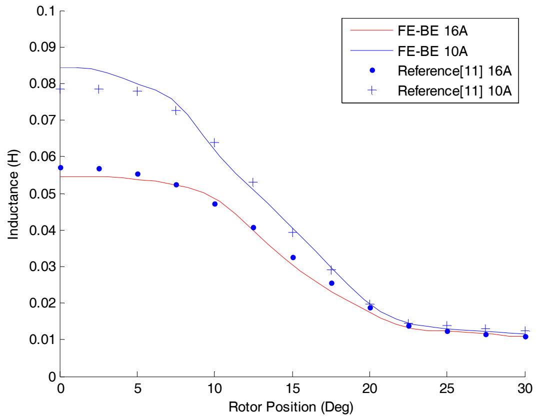

Fig. 5. Inductance of phase I versus rotor position for I= 10 A and I= 16 A.

function of the rotor position, and the phase current $L(\theta,i)$.

Fig (6) show that the higher the current the lower the inductance.

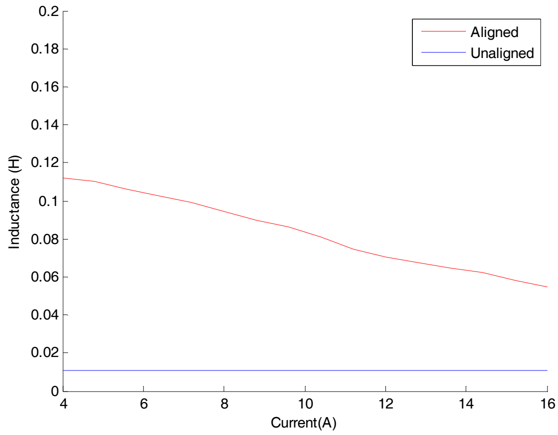

Fig. 6. Inductance of phase I versus phase current.

4.3. Static torque

Fig (7) shows that the torque is produced by the overlap between stator and rotor poles and has a minimum value (equilibrium) for the fully aligned and unaligned position of Fig (1) and Fig (2).

The comparison made for the inductances and the torques calculated by the proposed coupled FE-BE method with the prototype design values cited by references [11] show a good correlation and the small difference is mainly due to the great number of elements used in FLUX 2D compared with the number of elements used in our simulation.

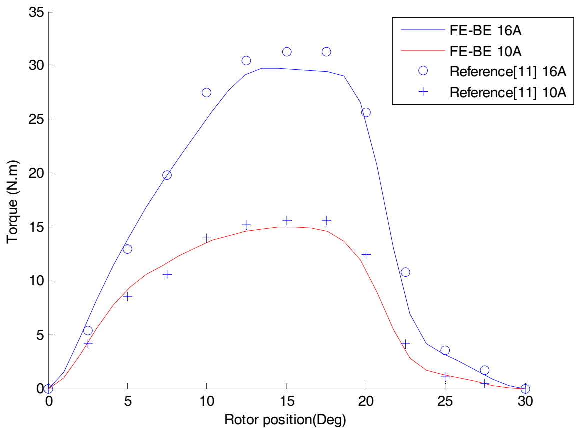

Fig. 7. Static torque versus rotor position for I= 10 A and I= 16 A.

5. CONCLUSION

The accuracy of the proposed coupled FE-BE method has enabled us to understand in more detail the magnetic flux distribution in an 8/6 SRM. It is a powerful tool used to compute important parameters such as flux linkage, inductance and static torque in the motor.

Numerous advantages are obtained with the coupled FE-BE method compared to the FEM method alone. Changing the position of the rotor in the coupled FE-BE method, does not require a new mesh and therefore the discretization is considerably simpler and less time consuming.

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