Abstract—The electric networks are considered as sinusoidal systems. The situation has changed significantly in recent years, as more and more harmonic-producing equipment is connected to the public distribution systems. Each non-linear load generates periodic events (e.g. harmonics) that could lead to serious problems within power system networks and its components.

As a result, it is necessary to investigate more detailed the effects of these new operation conditions on the components of the power systems, especially for power transformers.

If transformers operate in networks where the harmonic spectra exceed a certain limit, they will additionally be heated up and losses are increased, compared to operation under normal conditions.

In this paper is presented a method to determine the additional losses in transformers caused by harmonic currents. It is investigated the growth of no load losses, hysteresis losses and eddy current losses in magnetic iron-core and load losses, winding losses and stray losses. All these losses are considered in the evaluation of power transformers operating in harmonic polluted operation systems.

Index Terms—transformers, harmonics, no load losses, load losses.

I. NOMENCLATURE

\[ P_T \] Total losses in transformer, [W];
\[ P_0 \] No-load losses, [W];
\[ P_S \] Load losses, [W];
\[ \Delta P_{hys} \] Hysteresis losses, [W];
\[ \Delta P_{eddy} \] Eddy current losses in core, [W];
\[ \Delta P_{Rj} \] Losses due value of the current and resistance of the transformer, [W];
\[ \Delta P_{EC} \] Eddy Current Losses in windings, [W];
\[ \Delta P_{SL-R} \] Stray losses at rated current and frequency, [W];
\[ \Delta P_{EC-R} \] Eddy current loss at fundamental frequency, [W];
\[ f \] Frequency, [Hz];
\[ f_k \] Frequency factor;
\[ R_{Ip,s} \] Winding resistance (primary and secondary);
\[ k_{ol} \] Resistance increase factor;
\[ B_{max} \] Flux density in the core, [T];
\[ \mu \] Magnetic permeability of core material, [H/m];
\[ \rho \] Resistivity of core material, in [\(\Omega \cdot m\)];
\[ \gamma \] Density of core lamination, [kg/m^3];
\[ g \] Thickness of the lamination, [m];
\[ s_{col} \] Cross section of the core-legs, [m^2];
\[ V \] Volume of magnetic core, [m^3];
\[ d \] Diameter of round conductor, [m];
\[ a \] Width of rectangular conductor, [m];
\[ b \] Height of rectangular conductor, [m];
\[ m \] Number of conductors per coil height;
\[ n \] Number of conductors per coil width;
\[ h_B \] Coil height, [m];
\[ k_R \] Rogovski coefficient;
\[ S_N \] Rated power of transformer, [kVA];
\[ I_f \] Rated current, [A];
\[ I_h \] Current at harmonic order \( h \), [A];
\[ h \] Harmonic order, 1, 2, 3, etc.;
\[ \delta_I \] Total harmonic distortion for current;
\[ \beta \] Loading of transformer.

II. INTRODUCTION

The segment of loads causing harmonic pollution of power systems is increasing due to the rising volume of electronic and power electronic components. Each non-linear load generates periodic events (e.g. harmonics) that could lead to serious problems within power system networks and its components (e.g. transformers) [1].

Transformer simulation under sinusoidal operating conditions is a well-researched subject and many steady state and transient models are available.

The measurement and calculation methods, required by the standards, accurately determine a transformer’s losses and energy efficiency when it supplies linear resistive and/or...
inductive loads. The method used to determine total losses requires the summation of no-load losses and load losses. These losses are determined by performing an open-circuit and a short-circuit tests.

Unfortunately, modern electrical distribution systems typically supply a high percentage of non-linear electronic loads. As a result, transformer losses increase and energy efficiencies decrease. The level of deterioration is a function of harmonic voltage magnitudes at a transformer’s primary terminals, load-generated harmonic current magnitudes at its secondary terminals and their phase configurations. There are, unfortunately, no recognized standards for determining a transformer losses or efficiency under these non-linear conditions.

Transformers are designed to deliver the required power to the connected loads with minimum losses at fundamental frequency.

Harmonic distortion of the current, in particular, as well as of the voltage will contribute significantly to additional heating. To design a transformer to accommodate higher frequencies, designers make different design choices such as using continuously transposed cable instead of solid conductor and putting in more cooling ducts. As a general rule, a transformer in which the current distortion exceeds 5 percent is a candidate for derating for harmonics.

As a result, it is necessary to investigate more detailed the effects of these new operation conditions on the components of the power systems, especially for power transformers.

### III. Transformer Losses

An ideal transformer would have no power losses, and would be 100% efficient. In practical transformers energy is dissipated in the windings, core, and surrounding structures.

Distribution transformers are very efficient electrical machines reaching maximum efficiency at the level of 97.5% to 99.4%. Operating efficiency is smaller because transformers do not operate at maximum efficiency all the time.

This maximum efficiency point is at the point where load losses proportional to square of transformer load are equal to the no load losses which are constant and appear all the time when the transformer is energized (usually between 40% and 50% loading).

Transformer losses are produced by the electrical current flowing in the coils and the magnetic field alternating in the core. The losses associated with the coils are called the load losses, while the losses produced in the core are called no-load losses.

Transformer losses can be determined with the following mathematical expression:

$$ P_T = P_0 + P_S $$ (1)

A. Transformer no-load losses

No-load losses are caused by the magnetizing current needed to energize the core of the transformer, and do not vary according to the loading on the transformer. They are constant and occur 24 hours a day, 365 days a year, regardless of the load, hence the term no-load losses.

No-load losses (also referred to as excitation losses, core losses, or iron losses) are a very small part of the power rating of the transformer, usually less than 1%.

However, these losses are considered constant over the lifetime of the transformer (do not vary with load), and thus they generally represent a sizeable operating expense, especially if energy costs are high. Therefore, accurate measurements are essential in order to evaluate individual transformer performance accurately.

No-load losses are usually quoted and reported based on a sine-wave voltage excitation. Even with a sinusoidal source voltage, the non-linearity of the transformer core introduces significant harmonics into the excitation current and could result in distorted excitation voltage and flux waveforms. The magnitude of the voltage waveform distortion is usually determined by the output impedance of the voltage source and the magnitude and harmonics of the excitation current. The higher these parameters are, the greater will be the magnitude of the voltage waveform distortion [2].

No-load losses include losses due to magnetization of the core, dielectric losses in the insulation, and winding losses due to the flow of the exciting current and any circulating currents in parallel conductors.

1) Hysteresis losses

Hysteresis losses are caused by the frictional movement of magnetic domains in the core laminations being magnetized and demagnetized by alternation of the magnetic field. These losses depend on the type of material used to build the core. Silicon steel has much lower hysteresis than normal steel but amorphous metal has much better performance than silicon steel. Nowadays hysteresis losses can be reduced by material processing such as cold rolling, laser treatment or grain orientation [2].

Amorphous alloys differ from conventional crystalline alloys in their magnetic and mechanical properties. Amorphous metal core transformers improve electrical power distribution efficiency by reducing transformer core losses [3]. Amorphous technology is especially appropriate for transformers operating with low loading because the effect of harmonics on no-load losses is reduced.

Total area enclosed by the B-H loop is the measure of the hysteresis loss per unit volume per unit cycle. To reduce hysteresis loss one has to use a core material for which area enclosed will be as small as possible.

These losses can be calculated if some parameters (core dimensions) are known. The hysteresis losses can be estimate with the following mathematically expression:

$$ \Delta P_{his} = \frac{2 \cdot f \cdot \delta \cdot B_{max}^2}{\mu} \cdot V \ [W] $$ (2)

The growth of hysteresis losses is direct proportional with the frequency. If harmonic distortion exists in supply voltage of the transformer, than the losses will be increase. In next
figure it is presented the increase of hysteresis losses, for different types of transformers: cold rolling grain oriented transformers (CRGO) and amorphous transformers (AMTr). The rated power for both transformers is 2000kVA.

Hysteresis losses are usually responsible for more than a half of total no-load losses (50% to 70%). This ratio was smaller in the past (due to the higher contribution of eddy current losses particularly in relatively thick and not laser treated sheets).

2) Eddy current losses

Eddy current losses are caused by varying magnetic fields inducing eddy currents in the laminations and thus generating heat. These losses can be reduced by building the core from thin laminated sheets insulated from each other by a thin varnish layer to reduce eddy currents [2]. Using this technique, the magnetic core is equivalent to many individual magnetic circuits, each one receiving only a small fraction of the magnetic flux (because their section is a fraction of the whole core section). Furthermore, these circuits have a resistance that is higher than that of a non-laminated core, also because of their reduced section. From this, it can be seen that the thinner the laminations, the lower the eddy currents.

These losses can be estimated with the following mathematically expression:

$$\Delta P_{\text{eddy}} = \frac{\pi^2}{6 \cdot \rho \cdot \gamma} \cdot g^2 \cdot f^2 \cdot B_{\text{max}}^2 \cdot V [W]$$ (3)

The growth of eddy current losses is direct proportional with the square of frequency and square of thickness of laminations. The variation of eddy losses in the core with frequency is presented below. The same transformers are taking in consideration like above.

Eddy current losses nowadays usually account for 30% to 50% of total no-load losses. When assessing efforts in improving distribution transformer efficiency, the biggest progress has been achieved in mitigation of these losses.

The sum of hysteresis and eddy current losses is called core loss as both the losses occur within the core (magnetic material). For a given magnetic circuit with a core of ferromagnetic material, volume and thickness of the laminations are constant, the total core loss can be expressed as follows.

$$P_0 = \Delta P_{\text{his}} + \Delta P_{\text{eddy}}$$ (4)

The growth of core losses is presented in Fig. 3, for CRGO and AMDTr transformers.
B. Transformer load losses

These losses are commonly called copper losses or short circuit losses.

Transformer load losses include $I^2R$ losses in windings due to load current, eddy losses due to leakage fluxes in the windings, stray losses caused by stray flux in the core clamps, magnetic shields, tank wall, etc., and losses due to the flowing of current in parallel windings and parallel conductors within windings [4].

$$P_L = \Delta P_{RI} + \Delta P_{EC} + \Delta P_{SL}$$  \hspace{1cm} (5)

Load losses vary according to the transformer loading; they are composed of:

1) Ohmic losses

Ohmic heat losses sometimes referred to as copper losses, since this resistive component of load losses dominates. These losses occur in transformer windings and are caused by the resistance of the conductors [5]. The magnitude of these losses increases with the square of the load current and are proportional to the resistance of the windings. They can be reduced by increasing the cross-section of conductor or by reducing the winding length. Using copper as the conductor maintains the balance between weight, size, cost and resistance; adding an additional amount to increase conductor diameter, consistent with other design constraints, reduces losses.

The magnitude of the ohmic losses increases with the square of the load current and are proportional to the resistance of the windings [6].

$$\Delta P_{RI} = 3 \cdot R_{IP,i} \cdot I^2$$  \hspace{1cm} (6)

If the transformer works in harmonic polluted regime, the winding resistance (due to increase of a.c. factor) and the electrical current passing through winding will increase from pure sinusoidal regime. Such losses are determined as follows:

$$\Delta P_{RI} = 3 \cdot R_{IP,i} \cdot \sum_{i=1}^{n} k_{ca,i} \cdot I_{ip,i}^2$$  \hspace{1cm} (7)

This factor ($k_{ca}$) which increases the resistance of the transformer can be determined by different relations depending on the type of conductor in the winding as follows:

For round conductors:

$$k_{ca} = 1 + 0.8 \cdot \left( k_R \cdot \frac{d \cdot n}{h_B} \right)^2 \cdot k_f \cdot d^4 \cdot m_s^2$$  \hspace{1cm} (8)

The variation of this factor with frequency is presented in Fig. 4; the diameter of round conductor is 1.555 [mm].

For rectangular conductors

$$k_{ca} = 1 + 1.73 \left( k_R \cdot \frac{b \cdot n}{h_B} \right)^2 \cdot k_f \cdot a^4 \cdot \left( m_s^2 - 0.2 \right)$$  \hspace{1cm} (9)

For Z connection of winding with rectangular conductors:

$$k_{ca} = 1 + 1.73 \left( k_R \cdot \frac{h \cdot n}{h_B} \right)^2 \cdot k_f \cdot a^4 \cdot \left( 2m_s^2 - 20 \left( 2m_s^2 \right)^2 + 64 \right)$$

In this case, the variation of $k_{ca}$ is presented bellow, when the height and width of rectangular conductor are 10.6 and 2.36 [mm], respectively.

The factor $k_f$ it is calculated in respect of frequency and resistivity of the material, by following mathematical expression:

$$k_f = \left( \frac{f}{\rho \cdot 10^5} \right)^2$$  \hspace{1cm} (11)
If the variation with frequency of the electrical resistance ($R_{pf}$) is neglected (it will be the resistance for the fundamental harmonic) [7], power losses in copper can be determine with:

$$\Delta P_{RI} = 3 \cdot R \cdot I_f^2 \cdot \left(1 + \frac{\delta^2}{I_f^2}\right) \cdot \beta^2$$  \hspace{1cm} (12)

The variation of power losses is presented for a transformer which has rated power equal to 2000 kVA. Losses variation, as a function of the total harmonic distortion for currents, is presented below (the load of transformer have different values such as 60%, 80% and 100%) – Fig. 6.

![Fig. 6. Power losses as a function of Total Harmonic Distortion for current for different loads](image)

2) Eddy currents losses in windings

Eddy currents, due to magnetic fields caused by alternating current, also occur in the windings. Eddy current losses are hard to be evaluated; usually the transformers manufacturers specify this type of losses.

Typical eddy current losses, for the fundamental frequency are presented in Table 1 [8], taking in consideration the rated power of transformers, voltage level and type of transformer (dry or oil filled).

<table>
<thead>
<tr>
<th>Type</th>
<th>Power [MVA]</th>
<th>Voltage [V]</th>
<th>$P_{EC-R}$ [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dry</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\leq 1$</td>
<td>-</td>
<td>-</td>
<td>3-8</td>
</tr>
<tr>
<td>$\geq 1.5$</td>
<td>5 KV MT</td>
<td>-</td>
<td>12-20</td>
</tr>
<tr>
<td>$\leq 1.5$</td>
<td>15 KV MT</td>
<td>-</td>
<td>9-15</td>
</tr>
<tr>
<td>Oil-filled</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\leq 2.5$</td>
<td>480 V JT</td>
<td>-</td>
<td>1</td>
</tr>
<tr>
<td>$2.5-5$</td>
<td>480 V JT</td>
<td>-</td>
<td>1-5</td>
</tr>
<tr>
<td>$&gt; 5$</td>
<td>480 V JT</td>
<td>-</td>
<td>9-15</td>
</tr>
</tbody>
</table>

The eddy current loss depends on the square of the conductor dimension perpendicular to the leakage flux field.

$$\Delta P_{EC} = P_{EC-R} \cdot \sum_{h=1}^{h_{max}} \left[\frac{I_h}{I_f}\right]^2 \cdot h^2$$  \hspace{1cm} (13)

At the ends of the winding the flux field bends and the larger dimension of the rectangular conductor is perpendicular to a vector component of the leakage flux field. Equalizing the height of primary and secondary windings, which can be achieved with any winding design, reduces the concentrated eddy losses at the winding ends. However, the magnitude is still greater than the middle of the winding due to this bending of the leakage flux field. Reducing conductor size reduces the percentage eddy current loss but, increases the ohmic loss.

Reducing the cross-section of the conductor reduces eddy currents, so stranded conductors are used to achieve the required low resistance while controlling eddy current losses. Effectively, this means that the ‘winding’ is made up of a number of parallel windings. Since each of these windings would experience a slightly different flux, the voltage would be slightly different and connecting the ends would result in circulating currents which would contribute to losses. This is avoided by the use of continuously transposed conductor, in which the strands are frequently transposed to average the flux differences and equalize the voltage.

3) Stray losses [9]

Stray loss occurs due to the stray flux which introduces losses in the core, clamps, tank and other iron parts.

Stray loss may raise the temperatures of the structural parts of the transformer. For dry-type transformers increased temperatures in these regions do not contribute to an increase in the winding hot spot temperature. For liquid-immersed transformers, the stray loss increases the oil temperature and thus the hot spot temperature of the windings.

A simple method to calculate stray losses (in core, clamps, tank and other parts of iron) is in function of rated power of the transformer.

Losses in core, clamps and other iron parts of transformer can be calculated with next expression; these formula approximate stray losses:

$$\Delta P_{SL-R} = k_{cv} \cdot S_N$$  \hspace{1cm} (14)

$k_{cv}$ represent a coefficient which depends on rated power of transformers. This coefficient can be estimated with the following expression, for rated powers of transformers less than 10 MVA

$$k_{cv} = 0,1 + \frac{S_N}{20000}$$  \hspace{1cm} (15)

Stray losses are common to assume that it will vary as the square of the current times the frequency (harmonic order), as shown by:
\[ \Delta P_{SL} = \left( 0.1 + \frac{S_N}{20000} \right) \cdot S_N \cdot \sum_{h=1}^{20000} \left[ \frac{f_h}{f_f} \right]^2 \cdot h \]  

As a result, to reduce power losses, it is necessary to reduce the maximum power load of the transformer or to take extra care in the design stage. Reducing the maximum power load is a practice called “de-rating”. 

IV. CONCLUSIONS

A practical transformer will differ from an ideal transformer in many ways. For example the core material will have finite permeability, there will be eddy current and hysteresis losses taking place in the core, there will be leakage fluxes, and finite winding resistances. We shall gradually bring the realities one by one and modify the ideal transformer to represent those factors.

The effect of core loss is manifested by heating of the core and is a real power (or energy) losses.

In case of using AMDTr transformers, hysteresis losses will be decrease compare to CRGO transformers. The density of magnetic flux is lower (1.56 T) in AMDTr beside CRGO (2.04T).

Eddy current losses in core are larger in CRGO transformers, because the thickness of the lamination is bigger (0.23 mm) compare with AMDTr transformers, where the thickness is 0.025 mm.

In case of using AMDTr the loss reduction ranges from 65 to 90% compared with typical silicon steel based transformers used in Europe under different load conditions (linear or non-linear).

Because the losses are higher, the operating temperature of the transformer is higher and the lifetime is considerably shortened. Even moderately loaded transformers supplying IT loads will have much lower lifetimes than expected, unless proper precautions are taken.

The economic effects of harmonics are shorter equipment lifetime and reduced energy efficiency.

Equipment such as transformers is usually expected to last for 30 or 40 years and having to replace them in 7 to 10 years can have serious financial consequences.

V. REFERENCES