State of the Art in the Optimal Design of Electromagnetic Devices

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Abstract: The aim of this paper is to supply a bibliographic insight in recent developments in the area of electromagnetic devices optimizations. Current trends indicate a preference in multi-objective optimizations. Among the popular optimization methods are modern evolutionary algorithms such as: invasive weed optimization (IWO), particle swarm optimization (PSO), variations of genetic algorithms etc. In the final part is presented a case study of optimization with genetic algorithm of a LVDT via interfacing MATLAB and COMSOL Multiphysics in real-time.

Keywords: automated optimal design, multi-objective optimization, surrogate modeling, kriging, genetic algorithms, particle swarm optimization, linear variable differential transformer

1. INTRODUCTION

Searching for optimal designs stands at the heart of many major engineering branches. Some examples are optimization of airfoils in aerospace engineering [1], optimization of beams under stress in civil engineering, optimization of carbon nano-tubes in chemical engineering etc. As concerns electromagnetic devices, the field is broad and deals with optimizing devices such as: identifying current systems for solenoids which produce uniform magnetic fields [2], optimal design of electrical motors, e.g.: brushless DC wheel motors [3], optimal design of antennas, e.g.: earth orbiting satellites [4], optimal design of superconductors [5], optimizing microwave applications [6], optimization of electrode shapes in electrochemical systems [7], optimal design of diverse sensors and transducers, e.g.: linear variable differential transformer-LVDTs, topology optimization of magneto-mechanical systems [8] etc.

As computational resources are evolving at an accelerated pace it becomes easier to model, analyze and optimize diverse complex devices as computation time decreases. It is also advisable for certain computation intensive problems to port the optimization problem to a parallel/distributed computation model. A clear example of the benefits that ensue from this practice can be seen in [9]. Here researchers use the tabu search with grid enabled computing done via domain decomposition on a series of TEAM Workshop electromagnetic benchmark applications [10].

A review of differential evolution methods suitable for a multicore processor can be found in [11]. It is clear the need for a more extensive move from a CAD process to an automated optimal design (AOD) software packages. Two promising AOD modules are OptiNet from Infolytica [12] and modeFRONTIER from Esteco [13]. They are both commercial software and also deal with multi-objective optimizations. AOD is closely related to finite element method (FEM) for computation of the electromagnetic field. In [14] it is shown the preservation of the use of FEM models at each step of the optimization procedure. An interesting aspect is the development of automatic mesh refinement methods as the current solution gets closer to the optimum. See [15] for deterministic single optimization methods and [16] for evolutionary multi-objective optimizations.

2. EVOLUTION TO STOCHASTIC OPTIMIZATION METHODS

Optimization methods and algorithms fall into two main categories: deterministic and stochastic. Historically, the first optimization methods to be used were the deterministic ones. They deal mainly with classic single objective optimization problems where the objective functions are continuous and differentiable on the search space. The next step was to introduce probabilistic algorithms. These methods can be considered as generalizations of deterministic optimizations. The main difference as opposed to classical approaches is that this class of algorithms deals at some point with random variables which influence the outcome of the optimization scheme. Also they are derivative free methods. This proves to be...
safer because inaccuracies in computing the gradient of a numerically ill-conditioned objective function might originate a false minimum. With the use of a classical benchmark, the Loney solenoid, we can observe this effect [17]. These heuristic methods are based on the laws of probability and statistical evaluations and include evolutionary algorithms and statistic algorithms. The probability density function (PDF) expresses the evolution of a random vector in the $n_i$ dimensional space:

$$f(x, m, d) \approx \exp \left[-\sum_{i=1}^{n_i} \frac{(x_i - m_i)^2}{d_i^2}\right]$$

subject to:

$$\int f(x, m, d) \, dx = 1,$$

where $m_i$ and $d_i$ are mean value and dispersion of a random sample $x_i$, respectively. According to [18] the general algorithm behind evolutionary computing stays unchanged. The procedure generating a new solution may be written as the difference equation:

$$x(t_{k+1}) = s \left[ v \left( x(t_k) \right) \right],$$

with the initial solution $x(0) = x_0$, $x(t_k)$ the population at time $t_k$, while $v$ is an operator of random variation driven by the PDF, and $s$ is the selection operator driven by the fitness function.

Nowadays the emphasis is on stochastic algorithms, hence the numerous algorithms known to be effective and the continuous development in this area. Also there are many bio based ones like the ant colony optimization [19], swarm optimization, genetic algorithms, artificial immune systems [20, 21], cultural evolution [22] and so on.

The most popular type of algorithms in recent years belong to the genetic algorithm family. One notable example is the modified genetic algorithm called the breeder genetic algorithm (BGA). It is based on artificial selection similar to that used by human breeders. In [23] the BGA performance is demonstrated on a test suite of multimodal functions. A particular type of genetic algorithm is the genetic swarm optimization algorithm (GSO). This is a hybrid algorithm that combines exploring capabilities of the particle swarm optimization with the exploiting capabilities of the genetic algorithm. A wireless energy transfer optimization based on the GSO is proposed in [24]. Using the same algorithm, in [25] a novel meander-grooved polarization twist reflector is optimized for Ku-band applications. In [26] is presented the use of the classical genetic algorithm to optimize the design of a voice-coil actuator for optical image stabilization. The performance of the optimization is improved by using combined electromagnetic analysis to reduce computation time.

In recent years a preference can also be seen for the use of particle swarm optimization (PSO). This algorithm is based on the self-organizing behavior of social insects. This algorithm is applied in this paper for the optimal design of a brushless DC motor [28]. Another use of the PSO is for the optimization of height in layered circular-cylindrical dielectric lens antennas [28]. A similar algorithm to PSO has appeared recently. It is called invasive weed optimization (IWO), also inspired by nature, by the colonizing weeds. In [29] it is employed to optimize various antenna configurations. With the advent of so many options in choosing an optimization method or algorithm, the person who wants to employ them must remember the “no free lunch theorem”. This axiom states that no singular optimization method or algorithm can be the best one in all optimization engineering applications. At best, it can suite a class of optimization problems with good results as to solution accuracy and convergence speed. So it makes sense for that class of problems to choose an algorithm that outperforms other algorithms [30]. Generally speaking stochastic methods are seen as the number one choice in solving complex optimization problems. However we may yet see an emergence of some novel deterministic methods in solving certain engineering problems. For example, in [4] the geometric ellipsoid optimization method is deployed to optimize the design of reflector antennas for optimal coverage.

3. MULTI-OBJECTIVE OPTIMIZATION

Single objective optimizations, as suggested by the term are limited in scope. The unique solution obtained is considered to be the global solution. For a comprehensive look into single optimization techniques the reader is advised to consult [31]. The engineer identifies one problem area in a supposed electromagnetic device and optimizes it by minimizing/maximizing the considered design parameter. If there are more than one design parameters, single objective optimization can deal with them only in a sequential way and thus it cannot account for the influence of one parameter over the other if there are such influences. Almost all engineering designs require the optimization of conflicting objectives. For example if we want to optimize the magnetic induction of a permanent magnet by shape optimization and we consider as objective the minimization cost of the material and several geometric objectives we will have to choose a trade-off from a number of design solutions. The cost increases as the volume of the magnet increases. The set of solutions that are feasible considering design restrictions are optimal if they are an expression of all the objectives and no other solution in the search space is superior to them. They are known Pareto optimal solutions or the Pareto set. These solutions are found on the Pareto Front (PF).

Identifying the Pareto Front from a set of points in a multi-objective space is the most important and also the most time-consuming task in multi-objective
optimization. A well-known and used way of doing this task is with a non-dominated sorting algorithm: NSGA. For faster results it often bodes well to approximate the Pareto optimality front at first. Mathematically the Pareto optimality can be defined as follows. Let us consider, without loss of generality, a multi-objective maximization problem with \( m \) parameters and \( n \) objectives:

\[
\max x = f(x) = (f_1(x), f_2(x), f_3(x), \ldots, f_n(x)) \tag{4}
\]

where \( x = (x_1, x_2, x_3, \ldots, x_n) \in X \tag{5} \)

and \( y = (y_1, y_2, y_3, \ldots, y_n) \in Y \tag{6} \)

are tuple. A decision vector \( a \in X \) is said to dominate a decision vector \( b \in X \) if:

\[
\forall i \in \{1, 2, \ldots, n\} : f_i(a) \geq f_i(b) \wedge \\
\exists j \in \{1, 2, \ldots, n\} : f_j(a) > f_j(b) \tag{7}
\]

All decision vectors that are not dominated by any other decision vector are called Pareto-optimal. Sometimes designers need to know also the locally optimum solutions. These solutions are typically found near the PF as stated in [32]. By using a niching higher order evolution strategy (NES), a large number of these local solutions can be determined in a single optimization run. It is possible to evaluate more than one objective function (in parallel or in series) to approximate the PF. This method is demonstrated on a magnetic shunting design problem [33]. An example of using Pareto optimality based on generalized differential evolution (GDE3) for microwave filter design is shown in [34]. Sometimes in certain optimization problems the constraints and/or the objectives depend on time. In this case the Pareto front is also time dependent. This is the case of dynamic multi-objective optimization. An application on this promising subject is in [35]. An alternative to Pareto optimality is the game theory has proven itself viable in finding an optimal solution, corresponding to the Nash equilibrium of a bi-objective design problem [35].

The amount of publications in the area of evolutionary methods of multi-objective optimizations showed the superiority of them with respect to classical methods. The genetic algorithm was first to be tackled and came to have the most diverse family, having many variations from the original one. Some popular GAs are: vector-evaluated GA; multi-objective GA; niched Pareto GA; strength Pareto evolutionary algorithm; non-dominated sorting GA (NSGA). The first tries in multi-objective optimizations were done with a vector optimization shape design of a permanent magnet synchronous machine. This subject was revised recently in [36]. The applicability of multi-objective optimizations (MOO) is very high as shown in [37]; they are used in optimal design in both low frequency and high frequency devices. The main direction of MOO in computational electromagnetism is the development of cost-effective algorithms [38, 39].

4. APPROXIMATING OBJECTIVE FUNCTIONS WITH SURROGATE MODELS

A surrogate model is used to interpolate an objective function for computation time reduction for CPU intensive function evaluations. The condition is to have \( m_i + n_i \) sampled points and to use a set of additional basis functions \( F \), each centered on one out of \( n_i \) observations. In general, the predictor can be written as:

\[
p(x) = \sum_{i=1}^{m} b_i \psi_i(x) + \sum_{j=1}^{n} \beta_j (x - x_j). \tag{8}
\]

Where \( \psi_i \) are basis functions modeling a global trend, while the \( \beta \) dependent term can be viewed as a functional deviation; coefficients \( b_i \) and \( \beta_j \) are found by e.g. least-square fitting. In surrogate model based optimization, an initial surrogate is constructed using some of the available budget of expensive experiments and/or simulations. The remaining experiments or simulations are run for designs which the surrogate model predicts may have promising performance. The process usually takes the form of the following search/update procedure:

1. Initial sample selection (the experiments and/or simulations to be run);
2. Construct the surrogate model;
3. Search surrogate model (the model can be searched extensively, e.g. using a genetic algorithm, as it is cheap to evaluate);
4. Run and update experiment/simulation at new location(s) found by search and add to sample;
5. Iterate steps 2 to 4 until out of time or design ‘good enough’.

Depending on the type of surrogate used and the complexity of the problem, the process may converge on a local or global optimum or perhaps none at all [40].

A popular surrogate model used in multi-objective optimizations is kriging. Kriging adheres to the least squares estimation algorithms. Kriging estimates the value of an unknown real-valued function, \( f \), at a point, \( x^* \), given the values of the function at some other
points. A kriging estimator is said to be linear because the predicted value is a linear combination that may be written as:

\[ \hat{f}(x^*) = \sum_{i=1}^{n} \lambda_i(x) f(x_i). \quad (9) \]

The weights \( \lambda_i \) are solutions of a system of linear equations which is obtained by assuming that \( f \) is a sample-path of a random process \( F(x) \), and that the error of prediction:

\[ \varepsilon(x) = F(x) - \sum_{i=1}^{n} \lambda_i(x) F(x_i), \quad (10) \]

is to be minimized in some sense. For instance, the so-called simple kriging assumption is that the mean and the covariance of \( F(x) \) is known and then, the kriging predictor is the one that minimizes the variance of the prediction error.

Before the model can be constructed, a certain number of points need to be sampled. This initial set is called an experimental design, and the process of selecting suitable points is called design of experiments. Two commonly used techniques of experimental design are the Latin hypercube and the Hammersley sequence. In the context of statistical sampling, a square grid containing sample positions is a Hammersley sequence. In the context of statistical experimental design are the Latin hypercube and the experimental design.

Another DOE method was devised by Dr. Taguchi. We can see in [41] how it is applied together with a neural network approach to optimize the design of a tubular permanent-magnet motor (TLPM) for thrust characteristics improvement. In the paper a multi-objective design optimization is presented to improve force ripple, developed thrust, and permanent-magnet volume simultaneously. In the perspective of surrogate modeling almost all algorithms have two stages: first the surrogate model is fitted to the observed points and, then, a utility function is used to find the next search point. An example where a one stage algorithm was implemented, using both kriging and radial basis function models can be seen in [42]. Recently a hybrid one-then-two stage algorithm has been proposed [43]. It contains three steps: initialization, one-stage experimental design, and two-stage search. Multi-objective optimization methods using surrogate models can be divided into scalarizing and non-scalarizing. The former combines multiple objectives into a scalar preference function and then use one of the methods for single objective optimization.

An application of the non-scalarizing method in surrogate modeling for an electromechanical application is in [44].

5. CASE STUDY

The linear variable differential transformer (LVDT) is a type of electrical transformer used for measuring translational displacement. In this section it is detailed an optimization of the characteristic of a LVDT. The novelty is in the optimization technique. For the analysis tool COMSOL Multiphysics was chosen and for the search tool, MATLAB with its genetic algorithm tool. They are interlinked in a real-time client/server configuration. The device contains two secondary coils in opposition, one primary, a casing and of course the moving magnetic core.

As the magnetic core moves farther from the zero point, the linear response of the LVDT decreases. As it is intended to improve this characteristic, the goal is to increase the linearity range. The principal factors that influence the linearity include the ratio of the areas of the primary coil to the secondary coils and the length of the movable core. For this case study the design parameters chosen are the length of the secondary coils, \( p_1 \) and the length of the magnetic core \( p_2 \). Because the length of the casing is kept constant, by modifying the length of the secondary coils, the primary coil reacts and its length changes accordingly. The initial values of the design variables are \( \{ p_0 \}^T = \{1.5 \ 2.0\}^T \ cm. \)
The design vector \( p \) is subject to the following design constraints:

\[
0.5 \leq p_1 \leq 2.4
\]
\[
1.5 \leq p_2 \leq 3.0
\]

\[ (11) \]

The objective function is described by the following expression:

\[
f(\{p\}) = \sum_{i=1}^{q} (u_i^{c} - u_i^{fix})^2,
\]

\[ (12) \]

Where:

- \( q \) represents the number of different positions of the magnetic core;
- \( u_i^{fix} \) represents the imposed secondary output voltages in each “i” different positions of the movable magnetic core;
- \( u_i^{fix} \) represents the secondary output voltages, computed with COMSOL for each “i” different positions of the magnetic core.

The induced voltages in the secondary coil may be found from the finite element solution by integrating the potential solution:

\[
u_i^{c} = 4 \pi f \cdot \frac{N_{ai}}{S_1} \int_{S_2} r \cdot A \cdot dA,
\]

\[ (13) \]

Where:

- \( f \) is the frequency of the primary coil current excitation (1 kHz);
- \( S_1 \) is the area of the secondary windings;
- \( N_{ai} \) is the number of turns on the primary coil (100).

Therefore, the numerically computed secondary output voltage is given by the equation:

\[
u_i^{c} = u_i^{c} - u_i^{2} .
\]

\[ (14) \]

The results obtained with the GA shape optimization, are given in the following table together with some specific algorithm parameters:

<table>
<thead>
<tr>
<th>Table 1</th>
<th>GA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chromosome length L</td>
<td>2</td>
</tr>
<tr>
<td>Population size M</td>
<td>20</td>
</tr>
<tr>
<td>Crossover probabilities ( p_c )</td>
<td>0.8</td>
</tr>
<tr>
<td>Mutation probabilities ( p_m )</td>
<td>[1.0;0.99,..0]</td>
</tr>
<tr>
<td>Maximum number of iterations</td>
<td>100</td>
</tr>
<tr>
<td>Optimum value of the objective function ( f[V^2] )</td>
<td>( 1.79321 \times 10^{-11} )</td>
</tr>
<tr>
<td>Computational time [hours]</td>
<td>( \approx 6 )</td>
</tr>
<tr>
<td>The design variables values corresponding to the optimum: ( p_1 ) (cm)</td>
<td>0.0594</td>
</tr>
<tr>
<td>( p_2 ) (cm)</td>
<td>( 0.002 \times 10^2 )</td>
</tr>
</tbody>
</table>

By analyzing the data given in Table 2 we can conclude that the optimum characteristic shown in Figure 4 has improved its mean relative error by a factor of two.

<p>| Table 2 |
|---|---|---|---|---|---|</p>
<table>
<thead>
<tr>
<th>Position of the movable magnetic core (mm)</th>
<th>Imposed output voltages (mV)</th>
<th>Initial output voltages (mV)</th>
<th>Final output voltages - GA (mV)</th>
<th>Initial relative error [%]</th>
<th>Final relative error – GA [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>-4.15E-06</td>
<td>-5.00E-05</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>2.5</td>
<td>0.0029</td>
<td>0.0029</td>
<td>0.0034</td>
<td>0</td>
<td>17.24</td>
</tr>
<tr>
<td>5</td>
<td>0.0057</td>
<td>0.0056</td>
<td>0.0068</td>
<td>1.75</td>
<td>19.3</td>
</tr>
<tr>
<td>7.5</td>
<td>0.0086</td>
<td>0.0078</td>
<td>0.01</td>
<td>9.3</td>
<td>16.28</td>
</tr>
<tr>
<td>10</td>
<td>0.0115</td>
<td>0.0095</td>
<td>0.013</td>
<td>17.39</td>
<td>2.04</td>
</tr>
<tr>
<td>12.5</td>
<td>0.0144</td>
<td>0.0105</td>
<td>0.0156</td>
<td>27.08</td>
<td>8.33</td>
</tr>
<tr>
<td>15</td>
<td>0.0172</td>
<td>0.0109</td>
<td>0.0177</td>
<td>36.63</td>
<td>2.94</td>
</tr>
<tr>
<td>17.5</td>
<td>0.0201</td>
<td>0.0108</td>
<td>0.0192</td>
<td>46.27</td>
<td>4.48</td>
</tr>
<tr>
<td>20</td>
<td>0.023</td>
<td>0.0103</td>
<td>0.0199</td>
<td>55.22</td>
<td>13.45</td>
</tr>
</tbody>
</table>

The initial and the final design of the LVDT are shown with their magnetic potential representations (Fig. 5).

The optimization technique presented in this case study has proven itself as a robust one. The results obtained are enough proof to continue work to extend this technique to optimization of other electromagnetic devices.

6. CONCLUSIONS

As various optimization methods, techniques or algorithms were presented in this article we can form an opinion on the immensity and diversity of this research.
domain. By an in detail analysis of the recent literature we observe the dynamism of the optimal design of electromagnetic devices area and we can identify future viable research directions.

There is definitely a need for a better and clearer way of classification of all optimization methods and algorithms. The danger is not to reinvent already studied algorithms or methods as they are in great number. There is a big interest also in optimization theory, and current publications in this area struggle to keep pace with the research done. Further developments in the MO area should be better integrated in industry applications. It will become more and more so with the development of computer based optimization processes in CAE environment.

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