Hybrid thermo-electromagnetic model in the Drying Processes in a Microwave Field

O. MARGHITAS, F.I. HATHAZI, V.D. SOPRONI, Dan D. MICU and V. IANCU

Abstract: The paper presents aspects of numerical modelling of a drying process of a material (product) in a microwave field. To provide simulation results, first it is necessary to develop a hybrid model by coupling the electromagnetic field equations and heat transport equations together and then solve the mathematical model by using combined numerical methods. To complete the computation, using the discretization of the obtained equations, we use the Partial Element Equivalent Circuit (PEEC) method. The large non-symmetric sparse system it was efficiently solved with a complex version of the Conjugate Gradient Method, implemented in MathCad program. Was developed an algorithm based on coupling a Variable Time Step Finite Difference Method and Control Surface Method to solve the equations uniformly. It is shown an application of numerical modelling, using numerical techniques, for study of the heating of some dielectrics with losses, of parallelepiped shape, situated in a applicator, excited with energy through a wave guide. The paper has an applicative research character, the obtained results being of practical use.

Keywords: thermo-electromagnetic model, numerical methods, simulation

1. INTRODUCTION

Microwave heating Compared with conventional heating, has many advantages such as simultaneous heating of a material in its whole volume, higher temperature homogeneity, and shorter processing time. So, microwave drying may be used as an alternate technique for faster drying of materials with efficient utilization of time and energy. The objective of this study was to develop a mathematical model to simulate the drying condition of wheat seeds during drying in a microwave field.

Many researchers have studied the drying and heating process through a variety of mathematical models and numerical methods. [1–6] Related to temperature distribution inside materials in microwave field, exist some problems which must be considered. Although experimental studies and qualitative theoretical analysis are important, numerical simulations are indispensable for understanding the complex microwave heating process and also predict and control its behaviours. Taking into account the rapid software development of numerical computation techniques, it has been possible to simulate the microwave heating process also on a personal computer. [7-9]

The specific objective of this paper is to formulate a generalized mathematical model of the microwave drying process.

Generally, numerical modelling and simulation of microwave heating process is an analysis of multiphysical coupling. The mathematic model consists in a coupling of Maxwell’s equation (electromagnetic field) and the heat transport equation (thermic field). Taking into account that in the heat transport equation appears an nonhomogeneous term (heating source is which is provided by microwave dissipated power) the temperature variation during the heating process can cause changes in the complex permittivity formula. This could also cause perturbations in the space and time variation of the electromagnetic field.

To provide some simulation results, first it is necessary to create a hybrid model by coupling the two equations together and that solve the created mathematical model by using some numerical methods.

The proposed model for studying the microwave drying is based on some assumptions: is used a rectangular wave guide with perfect conductor walls, the microwave field propagates in the wave guide independently of the Oy direction, the electromagnetic field is assumed to be 2D (xOz plane), the effect of the container on the electromagnetic field is neglected; the magnetic permeability $\mu$ is approximated with its value $\mu_0$ in the free space.

2. ELECTROMAGNETIC FIELD MODEL

In our assumptions, the guiding structures of the electromagnetic waves are rectangular and have conductive boundaries. The knowledge of the distribution of the electromagnetic field in the
microwave structures makes possible the knowledge of the constant of propagation of the waves, of its dependence on frequency, of the propagation speed, of the phase and attenuation constant, etc. [10-13]

The microwave field propagates through an isotropic medium with electrical permittivity \( \varepsilon \), magnetic permeability \( \mu \), electric conductivity \( \sigma \), electric volume charge density \( \rho_e \), so the governing equation are given by the relations:

\[
\begin{align*}
\text{rot} \mathbf{H} &= \sigma \mathbf{E} + j \omega \mathbf{\epsilon} \cdot \mathbf{E} \\
\text{rot} \mathbf{E} &= -j \omega \mu_{0} \mathbf{H} \\
\text{div} \mathbf{H} &= 0 \\
\text{div} \mathbf{E} &= \frac{\rho_e}{\varepsilon}
\end{align*}
\]

(1)

where \( \mathbf{E} = E(x, y, z, t) \) is the electric field intensity and \( \mathbf{H} = H(x, y, z, t) \) - magnetic field intensity.

For microwave of TE\(_{10}\) mode, we have the following assumptions for the electric and magnetic field components vectors:

\[
E_x = E_y = H_y = 0
\]

So taking into account the above assumptions we could rewrite equations (1) in the following form:

\[
\begin{align*}
\mu_{0} \frac{\partial H_x}{\partial t} &= \frac{\partial E_y}{\partial z} \\
- \mu_{0} \frac{\partial H_y}{\partial t} &= \frac{\partial E_x}{\partial z} \\
\left( \frac{\partial H_z}{\partial x} - \frac{\partial E_x}{\partial y} \right) &= \sigma E_y + \epsilon \frac{\partial E_y}{\partial t} \\
\left( \frac{\partial H_z}{\partial y} - \frac{\partial E_y}{\partial x} \right) &= \sigma E_x + \epsilon \frac{\partial E_x}{\partial t}
\end{align*}
\]

The solutions \( (E, H) \) for this system of partial differential equations (PDE) are uniquely determined if we know exactly the permittivity, the sources, the initial conditions and the boundary conditions (usually not easy problem to solve).

So it is introduced two auxiliary functions, \( V \)-electrical scalar potential and \( A \)-magnetic vector potential, to determine the \( (E, H) \).

Taking into account the relations:

\[
\begin{align*}
\mathbf{H} &= \frac{1}{\mu_{0}} \cdot \text{rot} \mathbf{A} \\
\mathbf{E} &= -\text{grad} (V) - j\omega \mathbf{A}
\end{align*}
\]

(3)

it is obtained the mathematical differential model for the electromagnetic field:

\[
\begin{align*}
\Delta \mathbf{A} - \mu_{0} \epsilon \frac{\partial^2 \mathbf{A}}{\partial z^2} &= -\mu_{0} \frac{\mathbf{J}}{\epsilon} \\
\Delta V - \mu_{0} \epsilon \frac{\partial^2 V}{\partial z^2} &= \frac{\mathbf{J}}{\varepsilon}
\end{align*}
\]

(4)

The guiding of the electromagnetic waves through the microwave structures is realized through the close relation between the electromagnetic field of the wave on the one hand and the loads or currents on the boundaries of the structure, with certain conditions of reflection on these boundaries, on the other hand.

Taking into account the assumptions that we have perfectly conducting boundaries we have the following boundary conditions:

\[
E_x = 0, \quad H_y = 0
\]

where \( E_x \) is the tangential component of electric field vector and \( H_y \) is the normal component of magnetic field vector.

Was introduced also the boundary conditions along the interface between two different materials (i.e dielectric material surface and air):

\[
E_{i1} = E_{i2}, H_{i1} = H_{i2}, \quad D_{d1} = D_{d2}, \quad B_{d1} = B_{d2}
\]

Taking into account the forward and backward waves it could be a possibility to introduce in the model also the absorbing boundary conditions at both ends of the wave guide:

\[
\frac{\partial E_x}{\partial z} = \pm \frac{1}{\sqrt{\mu_{0} \varepsilon}} \frac{\partial E_x}{\partial z}
\]

(5)

The electric and magnetic field intensities oscillates, so we introduce also the incident wave due to the magnetron:

\[
\begin{align*}
E_x &= E_{x0} \cos \left( \frac{\pi z}{l} \right) \cos(\omega t) \\
H_y &= H_{y0} \cos \left( \frac{\pi z}{l} \right) \cos(\omega t) = \frac{E_{x0} \lambda_{0} \sqrt{\varepsilon_{0}}}{\lambda_{y0}} \cos \left( \frac{\pi z}{l} \right) \cos(\omega t)
\end{align*}
\]

where \( E_{x0} \) and \( H_{y0} \) are the initial value of the electric and magnetic field, \( l \) is the length of rectangular wave guide in the x-direction, and \( \lambda_{0} \), \( \lambda_{y} \) are the wave lengths of microwaves in free space respectively in rectangular guide.

3. THERMAL FIELD MODEL

The homogeneity of the electromagnetic field on the surface of the dielectric is influenced by its position inside the applicator. A non-homogeneous distribution of the field will lead to a non-homogeneous temperature distribution. Consequently, we need to study the thermal field. [10, 11]

Due to the complexity of the phenomena, the mathematical model was developed based on the following assumptions in the MW thermal modelling:

\( a) \) uniform initial temperature within products to be heated;

\( b) \) volume changes during heating are considered;

\( c) \) convective boundary conditions;

\( d) \) no chemical reactions occur in the material;

\( e) \) local thermodynamic equilibrium is assumed;

\( f) \) temperature profiles assumed, like electromagnetic field, to be 2D in xOz plane.

To describe heat transfer, a microscopic energy balance is proposed considering the internal heat generation due to microwave energy. The governing equations based on a volume average approach lead to
the following equation describing the drying process of a material:

\[- \nabla \cdot (k \cdot \text{grad}(T)) + C_m \rho_m \frac{\partial T}{\partial t} = \frac{\partial P_{em}}{\partial V} \tag{6}\]

where \(k\), \(\rho_m\), and \(C_m\) are the thermal conductivity, material density and specific heat capacity (usually taken as constants), \(T = T(x,y,z,t)\) is the absolute domain temperature, \(P_{em} = P_{em}(x,y,z,t)\) is the electromagnetic power generated by microwave absorption, \(V\) is the material volume.

The relation (6) can be expressed in terms of a generic product shape index, called \(GI\): (0 for slabs, 1 for infinite cylinders and 2 for spheres) [10], [12]

\[- \frac{\partial T}{\partial t} + k \frac{\partial^2 T}{\partial x^2} + C_m \rho_m \frac{\partial T}{\partial t} = \frac{\partial P_{em}}{\partial V} \tag{7}\]

where \(x\) is the axial or radial coordinate.

In order to solve the equation (7), the following assumptions for the initial and boundary conditions, valid in all spatial coordinates considered, were imposed: [10-13]

\(\bullet\) the initial condition, in order to determine an unique solution of equation (7):
\[T(x,y,z,0) = T_0(x,y,z)\] \[0 \leq x \leq l\] \tag{8}

\(\bullet\) the adiabatic boundary condition
\[\frac{\partial T(x,y,z,t)}{\partial n} = 0 \Rightarrow -k \frac{\partial^2 T(0,y,z,t)}{\partial x^2} = 0\] \tag{9}

\(\bullet\) the convective boundary condition at the material surface:
\[-k \frac{\partial T(l,y,z,t)}{\partial x} = \alpha(T - T_0)\] \tag{10}

where \(\alpha\) is the surface thermal transfer coefficient, \(T_0\) is the exterior domain temperature and \(l\) is the half thickness of the radius.

The space and time variation of the temperature can change the \(\varepsilon\), \(\mu\) and \(\sigma\) of the material. For nonmagnetic and nonconductive material, we can only discuss about the modification of \(\varepsilon\).

Taking into account that the response delay of the materials to external electromagnetic fields depends on the field frequency, electrical permittivity is treated as a complex function of the angular frequency \(\omega\), \(\varepsilon = \varepsilon(\omega)\) being defined by:

\[D(x,y,z,t) = \varepsilon \cdot \mathbf{E}(x,y,z,t) = (\varepsilon' - j\varepsilon'') \cdot \mathbf{E}(x,y,z,t)\]

For conductive material we can consider also resistive losses by taking into consideration in the imaginary part of the electric complex permittivity the electric conductivity \(\sigma\) [11]

\[\varepsilon = \varepsilon' - (\varepsilon'' + \sigma / \omega)\] \tag{11}

Having defined the complex permittivity, the absorption of microwave energy was calculated, considering an exponential decay of the microwave power absorption, by the following equation: [12]

\[P = P_0 e^{-2\tau d}; \quad P = \frac{\partial P_{em}}{\partial V}\] \tag{12}

where \(P\) is the internal volumetric heat generation of microwave energy, \(P_0\) is the surface power, \(d = l - x\), is the maximum distance measured from the surface.

The attenuation factor, \(\tau\), is a function of dielectric constant \(\varepsilon'\) and loss factor \(\varepsilon''\) (the real and imaginary part of the material complex permittivity \(\varepsilon\)):

\[\tau = \frac{\sqrt{2\pi}}{0.122} \left(\frac{\varepsilon}{\cos^{-1} \left(\tan \left(\frac{\varepsilon''}{\varepsilon'}\right)\right)}\right)\] \tag{13}

4. THERMO-ELECTROMAGNETIC MODEL

The most difficult aspect concerning mathematical modelling of the drying processes in microwave field, is that the solution of electromagnetic field problems should be coupled with thermal solution.

When solving the coupling problem there has to be taken into consideration a few aspects: [13]

- electric permittivity has a non-linear dependence on humidity and temperature of the material and on the work frequency;
- humidity of the material has a dependence of the thermal convection coefficient at surface, caloric capacity, thermal conductivity;
- boundary conditions depend on the water evaporation speed, water pressure vapours, on latent evaporation heat [15].

The inhomogeneous item \(P\) is determined by the electromagnetic power density that is dissipated in the nonmagnetic and nonconductive material due to its dielectric losses, and can be expressed by:

\[P = \frac{1}{2} \left(\mathbf{E} \cdot \frac{\partial \mathbf{E}}{\partial t} - \varepsilon_0 \varepsilon \mathbf{E} \cdot \frac{\partial \mathbf{E}}{\partial t}\right); \quad \varepsilon = \varepsilon' - j\varepsilon''\] \tag{14}

For steady state harmonic electromagnetic fields it is obtained:

\[P = 50\pi \sqrt{\left(\varepsilon'\right)^2 + \left(\varepsilon''\right)^2} \cdot \tan^{-1}\left(\varepsilon' / \varepsilon''\right)} \cdot \mathbf{E}^2\] \tag{15}

5. APPLIED NUMERICAL METHODS

If the geometric model is concerned, one-dimensional (1D) and two dimensional (2D) models are more generally used for analytical studies, and three-dimensional (3D) model is more close to practical applications.

Three iterative steps are used to solve the coupled problem electromagnetic and thermal field:
I. solving the equivalent developed model for electromagnetic field distribution;
II. computing the electromagnetic dissipated power;
III. updating the complex electric permittivity according to the temperature changes.

Numerical methods are used for general electromagnetic problems and give approximate solutions to electromagnetic field equations but also are
computationally demanding in terms of time and memory consumption. Instabilities associated with integral equations techniques in time domain are well-known.

Solving the systems of partial differential equations, from the electromagnetic field model is more difficult comparing with the heat transport model, because it includes two vector equations meanwhile the thermal equation is a scalar one.

To solve the electromagnetic field model, the first step was to rewrite the relation determined for electric field in the following form:

$$\mathbf{E} = \mathbf{E}_0 + \sum_{k=1}^{N_1} \alpha_k \mathbf{N}_k + \sum_{k=1}^{N_2} \beta_k \cdot (- \text{grad} \mathbf{V}_k)$$

(16)

where \(N_k\) are vector form functions with \(\text{rot}(N_k)\) linearly independent and \(N_k = 0\) on the boundary. After some mathematical manipulations, taking into account the existence of form/test functions and applying Neumann conditions, we obtain the system of \((N_1+N_2)\) equations capable to solve numerically the electromagnetic field model in the microwave applicator [14-15]:

$$\int_{\Omega} \mu \cdot \text{rot} \left( \mathbf{N}_k \right) \cdot \text{rot} \left( \mathbf{E} \right) d\Omega = \omega^2 \int_{\Omega} \mathbf{N}_k \cdot \varepsilon \mathbf{E} \cdot d\Omega$$

$$\int_{\Omega} \varepsilon \mathbf{E} \cdot \text{grad} \left( V_m \right) d\Omega = 0$$

(17)

where \(k = 1, 2, \ldots, N_1\) and \(m = 1, 2, \ldots, N_2\).

The problem which occurs is the choice of the functions \(N_k\) and \(V_m\). The most convenient way is to define the \(N_k\) functions by the nodal elements or the edge elements and the \(V_k\) functions only by nodal elements.

To complete the computation, using the discretization of the obtained equations, we apply the Partial Element Equivalent Circuit (PEEC) method. The method is based on Field Integral Equation (FIE) conversion to partial circuit elements. This is a full wave technique used for solving mixed electromagnetic field and circuit problems in both time and frequency domain. We obtain a very large, sparse, and complex system \((N_1+N_2)\) equations. The solution is found by developing an algorithm based on Modified Nodal Analysis Method, in which a sparse matrix is created, and implementing it in MathCad program. The large non-symmetric sparse system it was efficiently solved with a complex version of the Conjugate Gradient Method, implemented also in MathCad program. [16]

The system of partial differential equation which models the thermal problem was numerically solved, to calculate the material temperatures as a function of position and time with proper initial and boundary conditions. Was developed a hybrid algorithm based on coupling a Variable Time Step Finite Difference Method and Control Surface Method to solve the equations uniformly. A Combined Iteration Procedure was implemented in MathCad program to calculate the time step for each step.

When using a time dependent numerical methods, it is important to take into account the compatibility of the two time steps in the electromagnetic field equation and the heat transport equation. Because the variation speed of the electromagnetic field is much faster than the one of the temperature field, is better to apply the Time Scaling Factor Method which deals very efficiently with this difficulty. [11]

6. EXPERIMENTAL SETUP

The mixed microwave and hot air spurt drying equipment is using wave guides to transport the microwave energy from the generator to the applicator where the microwaves will be absorbed by the agricultural product. The main scheme of the laboratory equipment designed (at Faculty of Electrical Engineering and Information Science, University of Oradea) to dry the cereals granules under microwave radiation is presented in figure 1. This contains a hyper frequency generator (800W, 2.45GHz) of variable power, a circulator, a monomode cavity that acts as an applicator. [13, 15]

![Fig. 1. General scheme of the microwave. Laboratory setup.](image)

The microwave applicator used in this study consists in a continuous wave guide systems made of couple of small cavities in serial connection. The wave guide system consist in wave guides, where the agricultural seeds are placed on the in motion conveyor exposed to microwave radiation. Almost all part of the microwave energy is absorbed and the rest is absorbed by loads placed at the entry and exits gates of the drying system. This type of applicator can be used even without loads in good condition and no damages. [17]

The incidental and reflected power is performed continuously, thanks to a bi-couple, two hyper frequency-detecting diodes and two wattmeter’s. The temperature of the material (product sample) is measured with a thermometer, which does not inflict significant changes on the sample to be measured.

7. EXPERIMENTAL RESULTS

In order to detect the optimal functioning regime for the mixed microwaves and hot air spurt system where performed a serial of experimental runs.
The considerate monomode applicator has a parallelepiped form with the interior sizes of 109.22×54.6×300 mm. The experimental equipment is supplied from the power system at 220 V ± 5% and frequency of 50 Hz. All temperature maps obtain during drying process was captured by using the FLUKE Ti20 infrared camera, figure 2. [13]

![Fig. 2. Thermal map of the grain bed during drying process for the sunflower seeds dried in microwave field.](image)

In the present study case of analysis for treating agricultural seeds in microwave field, first step was an analytic calculation concerning applicator dimensioning and selection of the microwave generator parameters. [15]

After that were performed numerical simulations of the electromagnetic field within the applicator using the facilities offered by the numerical field analysis software Ansoft HFSS (High Frequency Structure Simulator). [13]

Comparative results for different steps of the analysis during drying process of agricultural products are presented in the following figures.

The effects of microwave power on the drying curve have been studied for the continuous microwave power flow, see figure 6.

![Fig. 3. Electric field distribution into the grain bed.](image)

![Fig. 4. Electric field distribution in complex measurements in the port and the surface of the dielectric.](image)

![Fig. 5. Poynting vector representation in dielectric (top view).](image)

![Fig. 6. Effect of microwave power on drying constant at sample thickness, T=10 mm.](image)

During the numerical simulation stage by using numerical analysis software was pursued to obtain a uniform distribution of the electromagnetic field in the dielectric's volume by modifying the position of the dielectric in the cavity. From the whole set of modelling and simulations that we conducted during our research, the most suggestive results were chosen.

8. CONCLUSIONS AND DISCUSSIONS

Microwave-drying process applied in food industrial applications has many advantages compared to the other conventional methods. In this work, a mathematical model was proposed based on the electromagnetic field equations and heat transfer differential equation with the initial and boundary
conditions required to define the system during microwave heating.

Remarkable progress has been achieved in recent years in the numerical simulation of microwave heating, which range from numerical modelling and computing to many applications in processing kinds of materials. We believe that the development will continue based on more refined models, faster numerical methods and more complete description of the complex permittivity of more materials.

The model is useful to understand the effect of system geometry on temperature profile and to obtain the location of hot and cold spots depending also on the shape of the product. The numerical model was validated using experimental data obtained in the laboratory. The developed model can be applied under conditions observed in the usual practice of microwave field applications and constitutes a decision-support aid to select the proper operating conditions to optimize technological processes.

The research conducted using the laboratory unit combines the modelling with the experimental results for finding the best combination between applied energy and the material particularity.

In the electromagnetic field distribution over the dielectric surface and in the thermal images obtained with the thermal camera, the temperature in seed bed in some points it has higher values that can damage the quality and germination of seeds.

With the help of the numerical simulation, the complex process of interaction between microwave and materials will be further understood, the critical parameters influencing the process will be identified, and a rapid design of optimized and controllable applicators will be provided.

REFERENCES

O. MARGHITAS
Dan D. MICU
V. IANCU
Faculty of Electrical Engineering
Technical University of Cluj-Napoca, Romania

F.I. HATHAZI
V.D. SOPRONI
Faculty of Electrical Engineering and Information Science
University of Oradea, Romania