Digital Synchronous Detectors Using Discrete Hilbert Transform

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***Abstract**: Digital synchronous detectors using Discrete Hilbert Transform are synchronous detectors in quadrature, numerical operators. A digital synchronous detector consists of a multiplier circuit and a low pass digital filter. The synchronous detector is activated by two numerical sequences, one input numerical sequence that carries the measurement information and a reference numerical sequence that sets the origin of phase. The numerical low pass filter has a very narrow bandwidth, in order to select, at the output, only the continuous or slowly variable component. The numerical sequence input comes from sampling the analog signal, a harmonic signal, information carrier. The numerical reference sequence is software generated and is equivalent to the numerical sequence of a harmonic signal of constant amplitude. The sensitivity of the synchronous detector is dependent on the phase of the reference sequence. Therefore, to obtain maximum sensitivity, the phase of the numeric reference signal can be adjusted by a controllable algorithm. The paper presents a short theory about synchronous analog detectors. Furthermore, algorithms are developed to achieve discrete synchronous detectors, in the form of numerical operators. The instruments of the Discrete Fourier Transform and of the Discrete Hilbert Transform are summarized.*

***Key words**: synchronous detection, Discrete Fourier Transform, Discrete Hilbert Transform, discrete time.*

1. INTRODUCTION

1.1. Analog synchronous detectors

Digital Signal Processing and related systems benefit today a collection of enhanced performance mathematical algorithms. Also, the manufacturers of integrated circuits and systems offer a growing variety of chip performance and highly complex software packages, tools encompassing discrete-time signal processing.

Inside the electrical measurement techniques, measurement information is embedded in amplitude or phase of an amplitude modulated signal. Modulation is for translating the useful signal spectrum of low frequencies domain – where the „1/f” noise is present - in the medium frequencies domain, where this noise does not appear.

Let us consider the information described by time constant amplitude or as a slowly varying function $X(t)$.

Recovery measurement signal is made through a synchronous detection operation (coherent detection). Synchronous detection may be performed by analog multiplication operations: direct multiplication or multiplication by chopping. In the present paper, synchronous detection method is treated in discrete time.

First, synchronous detection by analog multiplier is summarized. It involves two operations: the actual multiplication by an analog multiplier and the operation of low pass filtering. **Figure 1** illustrates the principle of synchronous detection by analog multiplication.

Be an information carrier signal $x(t)$ of the form:

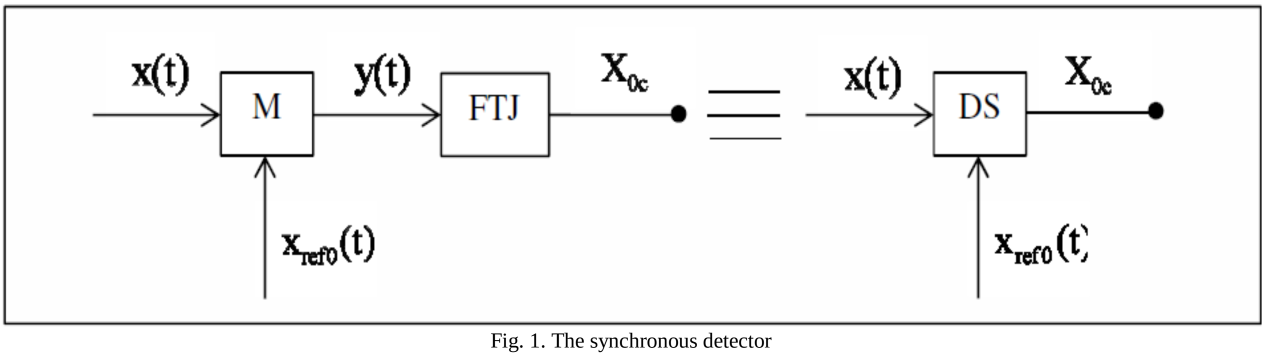

Fig. 1. The synchronous detector

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$$x(t) = X(t)\sin(\omega_0 t + \varphi) \quad (1)$$

where $X(t)$ is slowly variable, with the spectrum inside the low frequency domain; $\omega_0 = 2\pi f_0$ cu $f_0 =$ constant.

The reference signal is $x_{ref0}(t)$:

$$x_{ref0}(t) = X_{ref}\sin(\omega_0 t) \quad (2)$$

where X_{ref} = constant.

The synchronous detector “DS” comprises a multiplier „M” and a Low Pass Filter “FTJ”, figure 1:

Block "M" is the multiplier and multiplies the original signal by the reference signal and with a multiplier coefficient K :

$$\dim\{K\} = X^{-1}$$

1.2. Equations of the synchronous detector [1-11]

At the output of the multiplier, we obtain the signal $y(t)$:

$$y(t) = Kx(t)x_{ref0}(t) \quad (3)$$

$$y(t) = KX(t)\sin(\omega_0 t + \varphi)X_{ref}\sin(\omega_0 t)$$

$$y(t) = \frac{1}{2}KX(t)X_{ref}\cos\varphi - \frac{1}{2}KX(t)\cdot \cdot X_{ref}\cos(2\omega_0 t + \varphi) \quad (4)$$

where $(1/2)KX(t)X_{ref}\cos\varphi$ represents the slowly variable component, (or the DC component if $X(t)$ is constant) and:

$(1/2)KX(t)X_{ref}\cos(2\omega_0 t + \varphi)$ represents the AC component.

„FTJ” block is a Low Pass Filter which cuts the AC component and keeps the slowly variable component (or the DC component).

For the synchronous detector, a higher order (more than four) Butterworth filter type is chosen, in order to get a good rejection of alternative components of even frequency.

The FTJ output returns:

$$X_{0c} = \frac{1}{2}KX(t)X_{ref}\cos\varphi \quad (5)$$

where $X(t)$ contains the amplitude information and φ contains the phase information.

The block diagram of a synchronous detector is presented in figure 2.

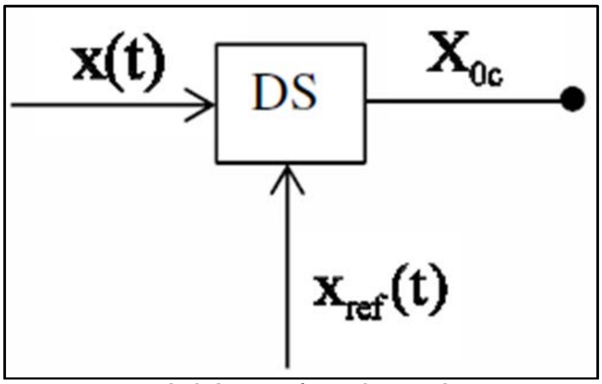

Fig. 2. Block diagram of a synchronous detector

1.3. Synchronous detector used to determine the amplitude information

In order to determine the amplitude $X(t)$, it is used a synchronous detector the reference voltage is out of phase with the same angle φ so that the two signals $x(t)$ and $x_{ref}(t)$ become in phase. This is realized with a phase shifter circuit, figure 3.

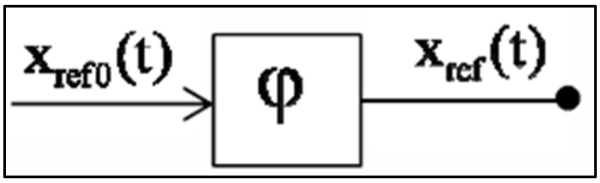

Fig. 3. Phase shifter circuit

The two signals applied to the synchronous detector are:

$$x(t) = X(t)\sin(\omega_0 t + \varphi)$$

$$x_{ref}(t) = X_{ref}\sin(\omega_0 t + \varphi)$$

At the output of the multiplier:

$$y(t) = \frac{1}{2}KX(t)X_{ref}\cos 0 - \frac{1}{2}KX(t)\cdot \cdot X_{ref}\cos(2\omega_0 t + 2\varphi)$$

At the output of the low pass filter we obtain:

$$X_{0c} = \frac{1}{2}KX(t)X_{ref} \quad (6)$$

1.4. Quadrature synchronous detector

It is used to determine also the phase information, see figure 4.

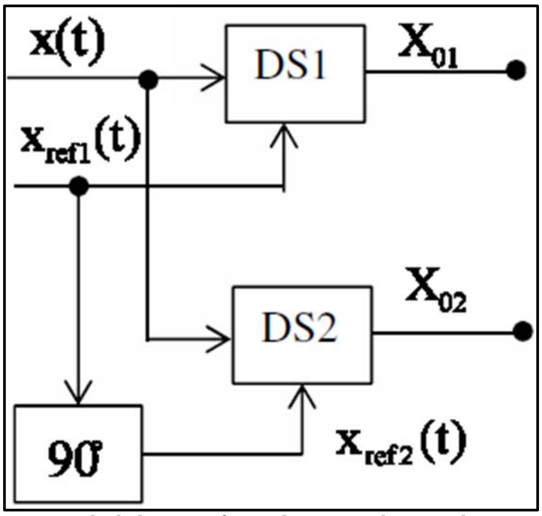

Fig. 4. Block diagram of a quadrature synchronous detector

The synchronous detectors DS1, DS2 are in quadrature – DS2 has its reference signal delayed with 90° to the reference signal of DS2. At the output of the synchronous detectors, we obtain the components X_{01} and X_{02}.

$$X_{01} = \frac{1}{2} K X(t) X_{ref} \cos(\varphi) \tag{7}$$

$$X_{02} = \frac{1}{2} K X(t) X_{ref} \sin(\varphi) \tag{8}$$

From the above relations, is obtained:

$$\varphi = arctg \frac{X_{02}}{X_{01}} \tag{9}$$

$$X_{01}^2 + X_{02}^2 = \frac{1}{4} K^2 X^2(t) X_{ref}^2 \tag{10}$$

$$X^2(t) = \frac{4(X_{01}^2 + X_{02}^2)}{K^2 X_{ref}^2} \tag{11}$$

2. SYNCHRONOUS DETECTORS IN DISCRETE TIME – NUMERICAL OPERATORS

Discrete signals are rows of data (discrete sequences) whose values were taken at various points in time. In order to be processed by a computing unit, discrete-time signal must be converted into a digital signal. Most often, this is done as part of the acquisition hardware circuit signals.

The following is a brief of two important tools in Digital Signal Processing, Discrete Fourier Transform TFD, and Discrete Hilbert Transform THD [8-12].

Given the signal *x(t)*, defined on the time-support [0, t_N], by uniform sampling with N samples, with the sampling period T_e, the discrete signal x[n] is formed:

$$x[n] = x(nT_e), \; n \in \overline{0, N-1},$$

where: $T_e = \frac{t_N}{N}$.

The sampling frequency f_e is chosen so that the Nyquist frequency $\frac{f_e}{2}$ to be greater than or equal to the least significant frequency from the *x(t)* signal's spectrum. Frequency discretization is made by taking the discretization step $f_0 = \frac{f_e}{N}$, respectively $\omega_0 = \frac{2\pi}{N} f_e$.

Discrete Fourier Transform TFD is :

$$TFD\{x([n]\} = X[k] = \sum_{n=0}^{N-1} x[n] e^{-jnk\frac{2\pi}{N}}, \quad k \in \overline{0, N-1} \tag{12}$$

Inverse Discrete Fourier Transform TFD^{-1} is:

$$TFD^{-1}\{X[[k]\} = x[n] = \frac{1}{N} \sum_{k=0}^{N-1} X[k] e^{jnk\frac{2\pi}{N}}, \quad k \in \overline{0, N-1} \tag{13}$$

Discrete Hilbert Transform applied to the sequence x[n] is:

$$H\{x[n]\} = \hat{x}[n] = TFD^{-1}\{\hat{X}[k]\} \tag{14}$$

where:

$$\hat{X}[k] = \begin{cases} -jX[k], & k = \overline{1, \frac{N}{2} - 1}. \\ & N \; even \\ jX[k], & k = \overline{\frac{N}{2} + 1, N - 1}. \end{cases} \tag{15}$$

We notice that are excluded: the DC and Nyquist components (for k = 0 and $k = \frac{N}{2}$), and for N odd:

$$\hat{X}[k] = \begin{cases} -jX[k], & k = \overline{1, \frac{N-1}{2}} \; . \\ & N \; odd \\ jX[k], & k = \overline{\frac{N+1}{2}, N-1} \; . \end{cases} \quad (15.a)$$

where the DC component is excluded.

Relations (15), (15.a) show the signal $\hat{x}[n]$ is out of phase with $-\pi/2$ to the original signal $x[n]$.

Inverse Discrete Hilbert Transform THD^{-1} is determined by the relation:

$$\mathbf{H}^{-1}\{\hat{x}[n]\} = -\mathbf{H}\{\hat{x}[n]\} \quad (16)$$

If *N*, the sequence length is an odd number and the DC component is missing, we can write:

$$\mathbf{H}^{-1}\{\hat{x}[n]\} = x[n]$$

If *N* is even, but the DC component is not equal to zero, or if N is odd - Nyquist component exists -, relation (16) is not strictly true.

2.1. Synchronous Detectors in Discrete Time

Synchronous detector in discrete time is illustrated in figure 5 [10]:

The block „FTJ numeric” represents a numerical Low Pass Filter. Digital filters are algorithms run on a computing unit.

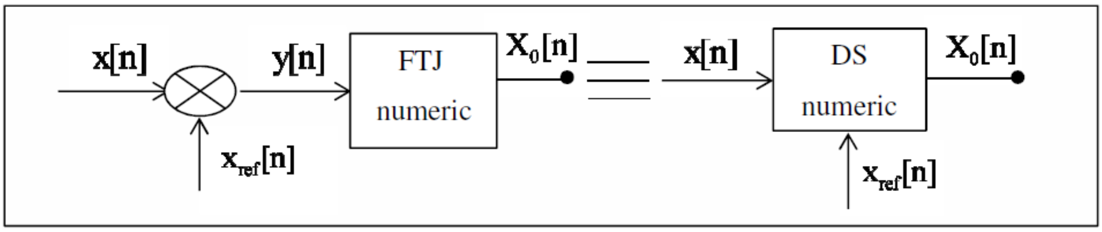

Figure 5. Block diagram of a discrete synchronous detector

2.1.1. How does the synchronous detector in discrete time operate?

It is considered known the sequences below:

$$x[n] = \{x[0]; x[1]; ...; x[N-1]\}$$

$$x_{ref}[n] = \{x_{ref}[0]; x_{ref}[1]; ...; x_{ref}[N-1]\}$$

Discrete signal $x[n]$ is arithmetically multiplied by reference signal to give:

$$y[n] = \{y[0]; y[1]; ...; y[N-1]\}$$

The obtained signal is passed through a numerical low pass filter. A new sequence of data is obtained, $X_0[n]$, which stabilizes at a certain value and contains the information of interest.

It is very important that the signals (data streams) applied to the synchronous detector to have the same length [13-15].

2.1.2. Discrete synchronous detector in quadrature. Equations of operation.

Synchronous detectors DS1 and DS2, in quadrature, have reference signals out of phase with $\pi/2$.

Be the input signals of the synchronous detectors:

$$x[n] = X[n]\sin[2\pi fn\Delta t + \varphi]$$

$$x_{ref1}[n] = X_{ref}\sin[2\pi fn\Delta t]$$

$$x_{ref2}[n] = X_{ref}\sin[2\pi fn\Delta t + \frac{\pi}{2}]$$

At the output of DS1 is obtained:

$$X_{01} = \frac{1}{2}X[n]X_{ref}\cos\varphi \quad (17)$$

At the output of DS2 is obtained:

$$X_{02} = \frac{1}{2}X[n]X_{ref}\sin\varphi \quad (18)$$

From relations (17) and (18) it can be determined $X[n]$:

$$X_{01}^2 + X_{02}^2 = \frac{1}{4}X^2[n]X_{ref}^2 \quad (19)$$

$$X^2[n] = \frac{4(X_{01}^2 + X_{02}^2)}{X_{ref}^2} \quad (20)$$

It can be obtained the phase φ, also:

$$\varphi = arctg\frac{X_{02}}{X_{01}} \quad (21)$$

2.2. Synchronous detector in quadrature using Hilbert Transform, by direct multiplication.

Hilbert Transform is used only in the case of discrete synchronous detection, for numerical sequences entries [14-18].

2.2.1. Synchronous detector in quadrature - Version I

Reference signals are in quadrature, figure 6.

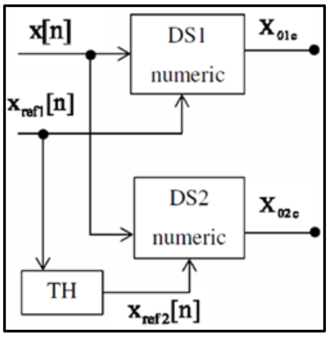

Fig. 6. Discrete synchronous detector in quadrature using Hilbert Transform – version I

The Hilbert transformer „TH" is used to out of phase with „$-\pi/2$" the reference signal 1 and to obtain the reference signal 2.

Be:

$$x[n] = X[n]\sin[2\pi fn\Delta t + \varphi_n]$$

Be the reference signal 1:

$$x_{ref1}[n] = X_{ref1}\sin[2\pi fn\Delta t]$$

The Hilbert conjugate of the reference signal 1 is determined and is obtained:

$$x_{ref2}[n] = \hat{x}_{ref1}[n] = -X_{ref1}\cos[2\pi fn\Delta t] = X_{ref2}$$

By direct multiplication of numerical sequences $x[n]$ and $x_{ref1}[n]$ is obtained also a numerical sequence:

$$x_{01}[n] = \frac{1}{2}X\ X_{ref1}\{\cos(\varphi_n) - \cos[2\pi fn\Delta t + \varphi_n]\}$$

By direct multiplication of numerical sequences $\mathrm{x[n]}$ and $\mathrm{x_{ref2}[n]}$ is obtained also a numerical sequence:

$$x_{02}[n] = \frac{1}{2}X[n]\ X_{ref1}\{\sin(\varphi_n) + \\ + \sin[2\pi fn\Delta t + \varphi_n]\}$$

The two resulting sequences are passed through a digital low pass filter and determine:

$$X_{01c} = \frac{1}{2}X[n]\ X_{ref1}\cos(\varphi_n) \quad (22)$$

$$X_{02c} = \frac{1}{2}X[n]\ X_{ref2}\sin(\varphi_n) \quad (23)$$

From the DC components, the amplitude and phase information are determined:

$$X^2 = \frac{4(X_{01c}^2 + X_{02c}^2)}{X_{ref1}X_{ref2}} = \frac{4(X_{01c}^2 + X_{02c}^2)}{X_{ref1}^2} \quad (24)$$

$$\varphi = arctg(\frac{X_{02c}}{X_{01c}}\frac{X_{ref1}}{X_{ref2}}) = arctg\frac{X_{02c}}{X_{01c}} \quad (25)$$

2.2.2. Synchronous detector in quadrature - Version II

Another interpretation of a synchronous detector in quadrature, with Hilbert, transformer, is illustrated in figure 7. Original discrete signal and its Hilbert conjugate are in quadrature, and the reference signal is the same for the two digital synchronous detectors.

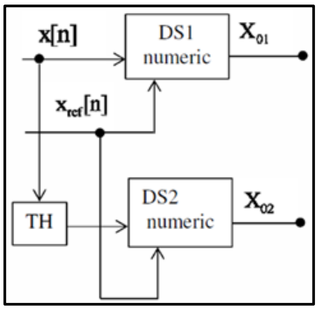

Figure 7. Discrete synchronous detector in quadrature using Hilbert Transform – version II

Be:

$$x[n] = X[n]\sin[2\pi fn\Delta t + \varphi_n]$$

If $X[n]$ has a spectrum in the low frequencies domain, then the Hilbert conjugate of the input signal is:

$$\hat{x}[n] = -X[n]\cos[2\pi fn\Delta t + \varphi_n]$$

Be the reference signal:

$$x_{ref}[n] = X_{ref}\sin[2\pi fn\Delta t]$$

By direct multiplication of the numerical sequences $x[n]$ and $x_{ref}[n]$ is obtained also a numerical sequence:

$$x_{01}[n] = \frac{1}{2} X[n]\ X_{ref}\ \{\cos(\varphi_n) - \\ - \cos[2\pi f n \Delta t + \varphi_n]\}$$

By direct multiplication of the numerical sequences $\hat{x}[n]$ and $x_{ref}[n]$ is obtained:

$$x_{02}[n] = -\frac{1}{2} X[n]\ X_{ref}\ \{\sin[2\pi f n \Delta t + \varphi_n] - \\ - \sin(\varphi_n)]\}$$

The two resulting sequences are passed through a digital low pass filter and determine the DC components:

$$X_{01c} = \frac{1}{2} X[n]\ X_{ref}\ \cos(\varphi_n) \quad (26)$$

$$X_{02c} = \frac{1}{2} X[n]\ X_{ref}\ \sin(\varphi_n) \quad (27)$$

From the DC components, the amplitude and phase information are determined:

$$X^2 = \frac{4(X_{01c}^2 + X_{02c}^2)}{X_{ref}^2} \quad (28)$$

$$\varphi = arctg\,\frac{X_{02c}}{X_{01c}} \quad (29)$$

3. CONCLUSIONS

Synchronous detection in real time is used in the processing of high frequency signals. Synchronous detection operations require high speed processing circuits, multipliers, analog switches, low pass filters, band pass filters.

The paper presents the basic principles of discrete-time synchronous detection using the Hilbert Transform.

Disclosed is a method for determining the amplitude and phase with two synchronous detectors in quadrature, using the Hilbert Transform, in two versions.

In the case of medium frequency and low frequency signals, is advisable discrete-time signal processing.

At the same time, in practical applications, synchronous discrete detectors are simpler to calibrate.

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