Use of Discrete Hilbert Transform for a Vector-Voltmeter in Discrete Time

R. A. Munteanu, G. Todoran

Faculty of Electrical Engineering, Technical University of Cluj-Napoca

Abstract *A digital synchronous detector consists of a multiplier circuit and a low pass digital filter. The synchronous detector is activated by two numerical sequences, one input numerical sequence that carries the measurement information and a reference numerical sequence that sets the origin of phase. The numerical sequence input comes from sampling the analog signal, a harmonic signal, information carrier. The sensitivity of the synchronous detector is dependent on the phase of the reference sequence. Digital synchronous detectors using Discrete Hilbert Transform are synchronous detectors in quadrature, numerical operators. The paper presents a short theory about synchronous analog detectors. Furthermore, algorithms are developed to achieve a synchronous detector in quadrature using Hilbert Transform with multiplication by chopping. Finally, the paper presents a general application to achieve a numerical vector-voltmeter, in two versions.*

Key words: *synchronous detection, Hilbert Transform, discrete time, vector-voltmeters*

1. INTRODUCTION

1.1. Analog synchronous detectors

Inside the electrical measurement techniques, measurement information is embedded in amplitude or phase of an amplitude modulated signal. Modulation is for translating the useful signal spectrum of low frequencies domain – where the „1/f" noise is present - in the medium frequencies domain, where this noise does not appear.

Let us consider the information described by time constant amplitude or as a slowly varying function $X(t)$.

Recovery measurement signal is made through a synchronous detection operation, called *coherent detection*. In the present paper, synchronous detection method is treated in discrete time, and is performed by analog multiplication by chopping.

Synchronous detection [1-6] involves two operations: the actual multiplication by an analog multiplier and the operation of low pass filtering. Figure 1 illustrates the principle of synchronous detection by analog multiplication.

Be an information carrier signal $x(t)$ of the form:

$$x(t) = X(t)\sin(\omega_0 t + \varphi) \qquad (1)$$

where $X(t)$ is slowly variable, with the spectrum inside the low frequency domain; $\omega_0 = 2\pi f_0$ with $f_0 =$ constant.

The reference signal is $x_{ref\,0}(t)$:

$$x_{ref\,0}(t) = X_{ref}\sin(\omega_0 t) \qquad (2)$$

where X_{ref} = constant.

The synchronous detector "DS" comprises a multiplier „M" and a Low Pass Filter "FTJ", figure 1:

Block "M" is the multiplier and multiplies the original signal by the reference signal and with a multiplier coefficient K :

$$\dim\{K\} = X^{-1}$$

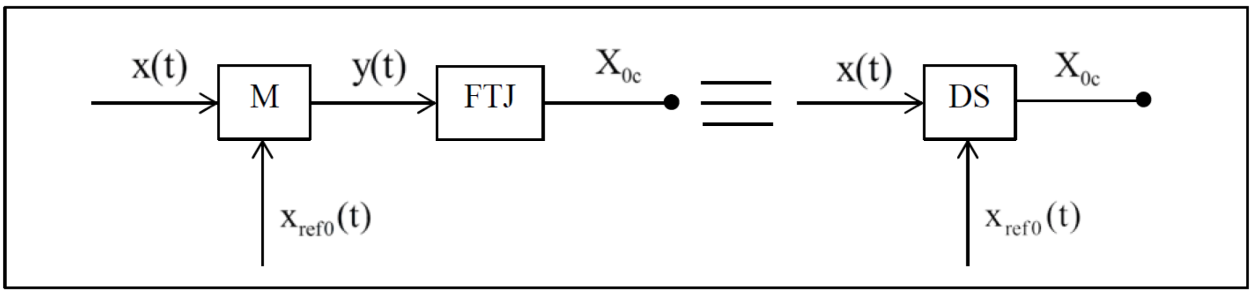

Fig. 1. The synchronous detector

At the output of the multiplier, we obtain the signal $y(t)$:

$$y(t) = Kx(t)x_{ref\,0}(t) \tag{3}$$

$$y(t) = KX(t)\sin(\omega_0 t + \varphi)X_{ref}\sin(\omega_0 t)$$

$$y(t) = \frac{1}{2}KX(t)X_{ref}\cos\varphi - \frac{1}{2}KX(t)X_{ref}\cos(2\omega_0 t + \varphi) \tag{4}$$

where $(1/2)KX(t)X_{ref}\cos\varphi$ represents the slowly variable component, (or the DC component if $X(t)$ is constant) and :

$(1/2)KX(t)X_{ref}\cos(2\omega_0 t + \varphi)$ represents the AC component.

„FTJ" block is a Low Pass Filter which cuts the AC component and keeps the slowly variable component (or the DC component).

For the synchronous detector, a higher order (more than four) Butterworth filter type is chosen, in order to get a good rejection of alternative components of even frequency.

The FTJ output returns:

$$X_{0c} = \frac{1}{2}KX(t)X_{ref}\cos\varphi \tag{5}$$

where $X(t)$ contains the amplitude information and φ contains the phase information.

The block diagram of a synchronous detector is presented in figure 2.

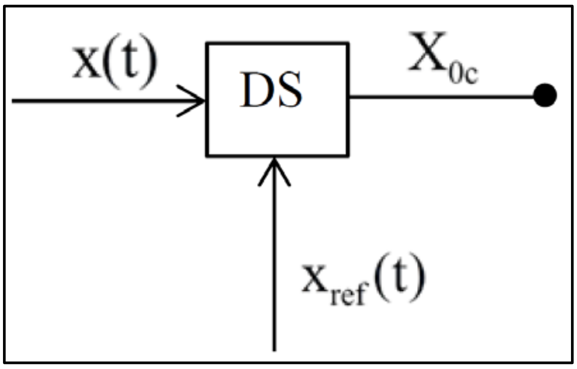

Fig. 2. Block diagram of a synchronous detector

1.2. Quadrature synchronous detector

It is used to determine also the phase information, see figure 3.

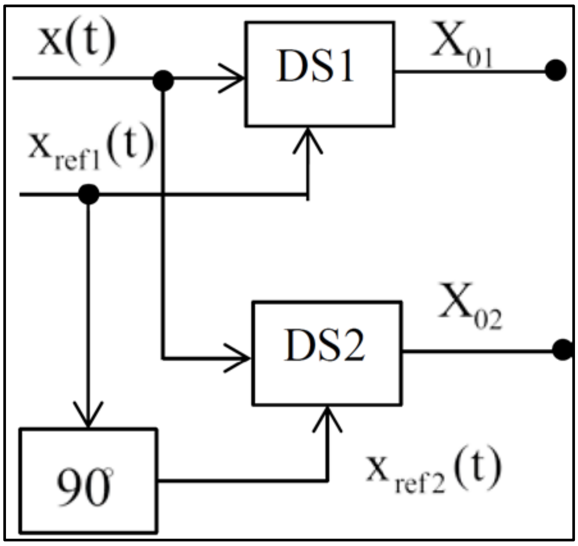

Fig. 3. Block diagram of a quadrature synchronous detector

The synchronous detectors DS1, DS2 are in quadrature – DS2 has its reference signal delayed with 90° to the reference signal of DS2. At the output of the synchronous detectors, we obtain the components X_{01} and X_{02}.

$$X_{01} = \frac{1}{2}KX(t)X_{ref}\cos(\varphi) \tag{6}$$

$$X_{02} = \frac{1}{2}KX(t)X_{ref}\sin(\varphi) \tag{7}$$

From the above relations, is obtained:

$$\varphi = arctg\frac{X_{02}}{X_{01}} \tag{8}$$

$$X_{01}^2 + X_{02}^2 = \frac{1}{4}K^2X^2(t)X_{ref}^2 \tag{9}$$

$$X^2(t) = \frac{4(X_{01}^2 + X_{02}^2)}{K^2X_{ref}^2} \tag{10}$$

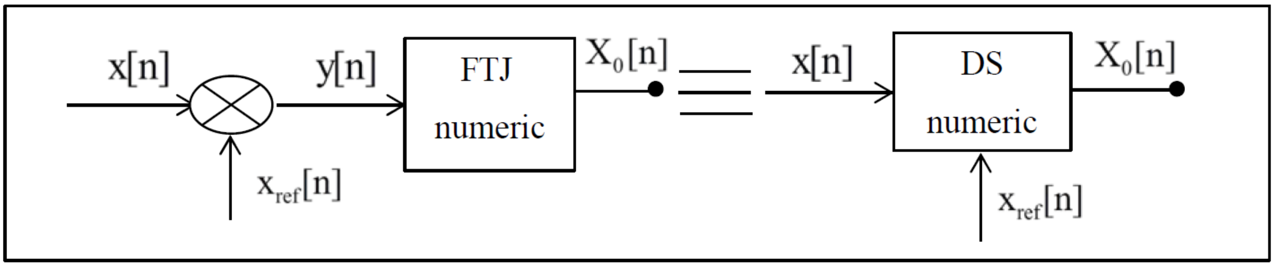

Fig. 4. Block diagram of a discrete synchronous detector

2. SYNCHRONOUS DETECTORS IN DISCRETE TIME

Synchronous detector in discrete time is illustrated in figure 4 [7]:

The block „FTJ numeric" represents a numerical Low Pass Filter. Digital filters are algorithms run on a computing unit.

2.1. How does the synchronous detector in discrete time operate?

It is considered known the sequences below:

$$x[n] = \{x[0]; x[1]; ...; x[N-1]\}$$
$$x_{ref}[n] = \{x_{ref}[0]; x_{ref}[1]; ...; x_{ref}[N-1]\}$$

Discrete signal $x[n]$ is arithmetically multiplied by reference signal to give:

$$y[n] = \{y[0]; y[1]; ...; y[N-1]\}$$

The obtained signal is passed through a numerical low pass filter. A new sequence of data is obtained, $X_0[n]$, which stabilizes at a certain value and contains the information of interest.

It is very important that the signals (data streams) applied to the synchronous detector to have the same length [9-11].

2.2. Synchronous detector in quadrature using Hilbert Transform with multiplication by chopping.

Synchronous detector in discrete time in quadrature – chopping version

Synthesis algorithm of synchronous detector in quadrature by chopping is based on considerations of analogue synchronous detector by chopping [8, 12-13]. In figure 5 is presented its general scheme.

The numerical sequence associated to input signal $x[n]$ is obtained in analogy with the above presented ones.

Reference signal $x_{ref1}[n]$ is obtained through signum operator {}:

$$x_{ref1}[n] = sign\{X_{ref1}\sin[2\pi fn\Delta t]\} = \frac{4}{\pi}\sum_{k=0}^{\frac{N-1}{2}} \frac{\sin(2k+1)\omega_0 nT_e}{2k+1}, \quad \left(n = \overline{0, N-1}\right) \tag{11}$$

Equations of synchronous detector DS1 are:

$$y_1[n] = \frac{4}{\pi} X(nT_e)\sin(nT_e + \varphi) \cdot \sum_{k=0}^{\frac{N-1}{2}} (\frac{\sin(2k+1)\omega_0 nT_e}{2k+1} \tag{12}$$

Thus, at the output of the synchronous detector is obtained the sequence:

$$X_{01c}[n] = \frac{2}{\pi} X[n]\cos\varphi[n] \tag{13}$$

Ie the multiplication coefficient of the synchronous detector is $\frac{2}{\pi}$.

The synchronous detector DS2 has the reference voltage:

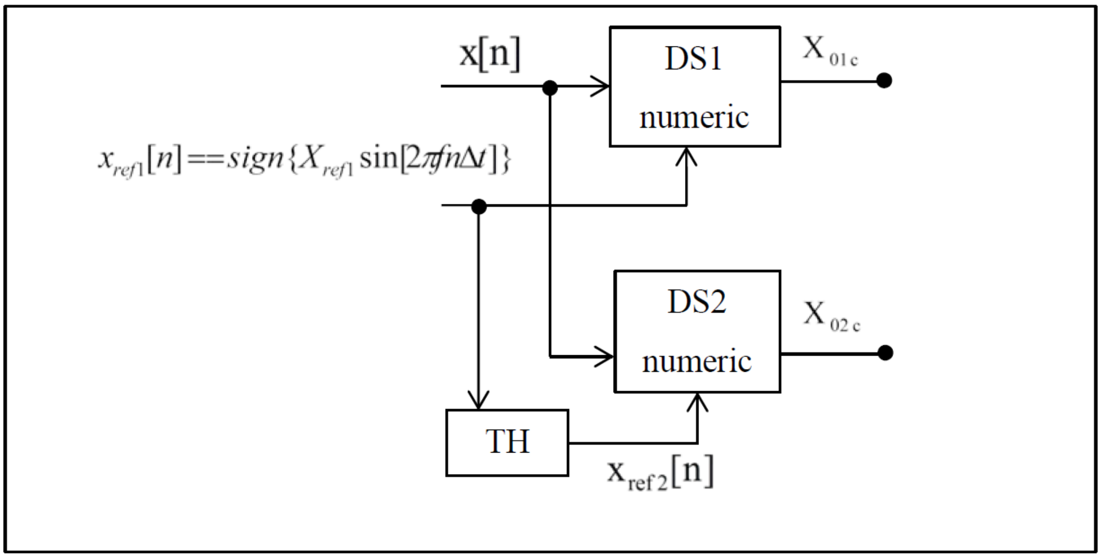

Fig. 5. Synchronous detector in discrete time in quadrature – chopping version

$$x_{ref\,2} = H\{x_{ref\,2}[n]\} = \hat{x}_{ref\,2}[n]$$
$$= \frac{4}{\pi}\sum_{k=0}^{\frac{N-1}{2}}(-1)^k \frac{\cos(2k+1)\omega_0 nT_e}{2k+1} \tag{14}$$

The equations of the synchronous detector DS2 are:

$$y_2[n] = \frac{4}{\pi} X(nT_e)\sin(\omega_0 nT_e + \varphi)\cdot$$
$$\cdot\sum_{k=0}^{\frac{N-1}{2}}(-1)^k \frac{\cos(2k+1)\omega_0 nT_e}{2k+1} \tag{15}$$

Thereby, at the output of the synchronous detector is obtained the sequence:

$$X_{02c}[n] = \frac{2}{\pi} X[n]\sin\varphi[n] \tag{16}$$

The digital filter selects the original signal if the cutoff frequency satisfies the condition: $\underline{\omega}_t < 2\underline{\omega}_0 = 2\omega_0 T_e = 2\pi\frac{f_0}{f_e}$

It is important to note that the input signal must have its spectrum's maximum frequency $\omega_M < \omega_0$. With this restriction fulfilled, the sequence y[n] has the spectrum composed of the original signal's spectral packages, placed in the origin and in the right of even frequencies, without such spectra intersects.

Observations:

The synchronous detector in discrete time by chopping has the advantage that it provides at the output sequences independent of the amplitude X_{ref}. This makes possible that on the primary measurement circuit the operation of calibration in amplitude of the reference signal not to be necessary.

On the other hand, in situations where the frequency ω_0 is accurately known (implicitly $\underline{\omega}_0$), the reference signal can be generated virtual. Such a situation may be accepted when the phase information is not important, but only the amplitude information of the analog signal $x(t)$.

Numerical output filters should have a strongly rejecter character. It is important to know the phase characteristics of the low pass filters.

3. APPLICATION VECTOR-VOLTMETER

3.1. Version I:

The Vector-voltmeter is a direct application to a configuration of two quadrature synchronous detectors. It is used to determine both the information contained in the amplitude and phase.

The block scheme of a Vector-voltmeter is illustrated in figure 6. The equations of the synchronous detector in quadrature – version I – are implemented.

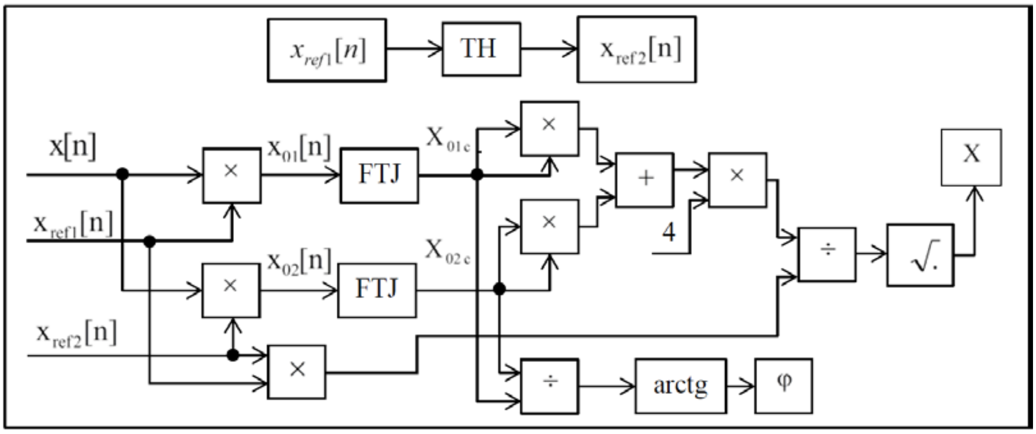

Fig. 6. Block scheme of a Vector-voltmeter - Version I

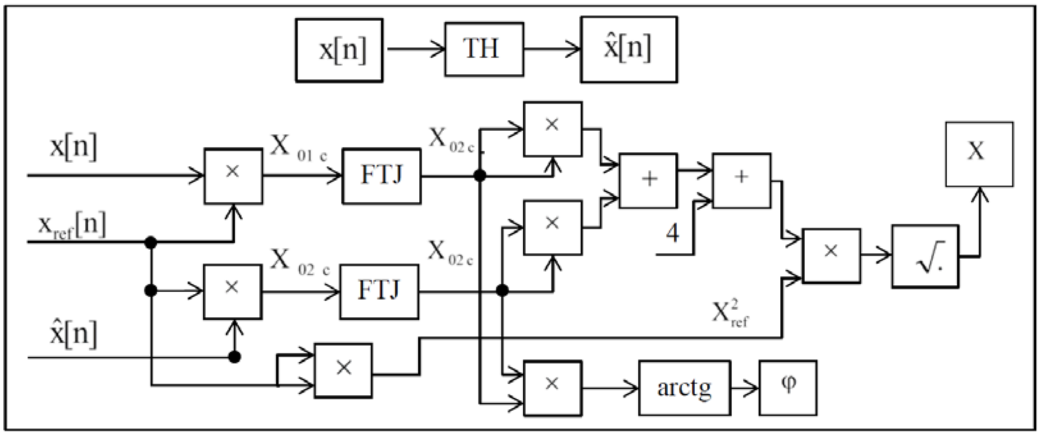

Fig. 7. Block scheme of a Vector-voltmeter - Version II

3.2. Version II:

The block scheme of a Vector-voltmeter is illustrated in figure 7. The equations of the synchronous detector in quadrature – version II – are implemented.

4. CONCLUSIONS

Synchronous detection in real time is used in the processing of high frequency signals. Synchronous detection operations require high speed processing circuits, multipliers, analog switches, low pass filters, band pass filters.

The paper presents a version of a discrete synchronous detector in quadrature by chopping, using the Hilbert Transform.

Also, two versions to achieve a Vector-voltmeter are presented.

Theoretical conclusion follows: if the measurement signal is embedded in the amplitude or phase, its recovery is preferred by an operation of synchronous detection in quadrature realized in discrete time, using the Hilbert Transform.

REFERENCES

1. Andrade, M., Messina, A. R. (2005), „Application of Hilbert Techniques to the Study of Subsynchronous Oscillations”, International Conference on Power Systems Transients (IPST’05) in Montreal, Canada, No. IPST05 - 172.
2. Français, O. (2003), „Detection synchrone”, Ecole S Superieure d’Ingenieurs en Electronique et Electrotechnique, Paris.
3. Kschischang, F.R. (2006), “The Hilbert Transform”, The Edward S. Rogers Sr. Department of Electrical and Computer Engineering, University of Toronto.
4. Todoran, Gh., Munteanu, R. jr. (2006), „Switch Function Used in Modulation and Syncronous Detection Operations. Shunt (by-pass) Type Switch Schemes”, Acta Electrotehnica, volume 47,no.3.
5. Todoran, Gh., Munteanu, R. jr. (2006), „Switch Function Used in Modulation and Syncronous Detection Operations. Series and Series-Shunt Type Switch Schemes”, Acta Electrotehnica, volume 47, no.3.
6. Todoran, Gh., Munteanu, R. jr., Holonec, R., Copandean, R. (2007), „Measurement of Series Circuit Parameters Z, R, L, C by Multiple Synchronous Detection Method”, Acta Electrotehnica, volume 48,no.1, pp. 64-70.
7. Todoran, Gh., Munteanu, R. jr., Rodica Holonec, R. Copandean. (2007), „Measurement of Parallel Circuit Parameters B, G, L, C by Multiple Synchronous Detection Method”, Acta Electrotehnica, volume 48,no.1, pp. 70-76.
8. Todoran, Gh., Munteanu, R. jr., Holonec, R., Copandean. R., Munteanu, M. (2007), „Measurement Methods of Circuit Parameters by Virtual Multiple Synchronous Detection Method”, Conferinta REV2007, Porto, 25-27 iunie.
9. Todoran, Gh., Holonec, R. (2007), „Cubic Spline Virtual Instruments Associated to Symbolic Transmittance of Circuit”, 15th IMEKO TC 4 Symposium on Novelties in Electrical Measurements and Instrumentations, Iasi 19-21 septembrie.
10. Todoran., Gh., Tarnovan, I., Tugudean, D. (2008), „Global Measurement Uncertainty”, Acta Electrotehnica, volume 49,no.3, pp. 385.
11. Todoran, Gh., Munteanu, R. jr., Copandean, R. (2008), „Multiple Synchronous detection method for meassuring non liniar and static resistance. Proceeding of the 11-th International Conference on optimization of electrical and electronic equipement”, Brasov mai 22-23, pp. 93-96.
12. Todoran, Gh. (2010), „Considerations About Discrete Time Analysis of Measuring Signals Using Discrete Hilbert Transform”, Acta Electrotehnica,volume 51,no.1, ISSN 1841-3323, pp. 76-82.
13. Todoran, Gh., Munteanu, R. jr. (2011), „Synchronous Detection Technique for Multiphase Voltage and Currents System Phasors Alignment Method”, Acta Electrotehnica, volume 52,no.4, ISSN 1841-3323, pp. 227-231.

Conf. dr. ing. Radu Adrian Munteanu
Prof. dr. ing. Gheorghe Todoran

Faculty of Electrical Engineering, Technical University of Cluj-Napoca, 26-28, G. Barițiu st., Cluj-Napoca, Romania
Radu.A.Munteanu@ethm.utcluj.ro
Gheorghe.Todoran@ethm.utcluj.ro