

Electromagnetic Field Numerical Modeling using BEM2D

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Abstract – Involving numerical modeling programs in the solving of engineering problems has reached a special amplitude in the last few years, thus it can be said that it became indispensable in investigating electromagnetic phenomena. So, in this paper the software program BEM2D elaborated by the authors in order to numerically model electromagnetic devices is presented. In order to highlight the complexity and utility of this program a simple example is presented at first, followed by the determination of its calculation accuracy through the presentation of an example solved with the implemented program, with a commercial software program and last but not least through analytical calculations. Final conclusions end this paper.

Keywords – boundary element method, numerical modeling, electromagnetic devices, BEM2D.

1. INTRODUCTION

Boundary element method (BEM) proves to be, through its advantages, one of the most efficient modeling and simulation methods for the behavior of fields in their different operating regimes. The study of the special literature shows that the extent of use of this method is currently growing, each year new domains of activity in which this method is successfully applied appearing [1], [2]. Due to this reason the authors chose the use of the Boundary element method for the realization of the presented numerical modeling software program.

The main idea of each numerical modeling technique is the domain and/or boundary discretization in a given number of elements.

In BEM the domain's boundary represents the only geometry which must be meshed. Of course, in the case where there are integral domain terms due to the sources from the interior of the work region the necessity of their numerical evaluation appears and hence the division of the domain to the mesh cells level is necessary. However, as shown in the special literature, that division in small sub-regions is based on another principle and another comparative purpose from what is meant by „domain discretization” in the case of methods of „domain” type like Finite element method (MEF) or Finite difference method (MDF) [2]. Boundary discretization in BEM is made using interconnected elements for which the geometry and physical variables variation modules must be specified for each element. These variations are in general of

polynomial type (constant, linear, square or of superior order) but can be also described by special functions.

Following the study of the special literature it was found that the specialists in the BEM and FEM domains have reached the conclusion that the use of isoparametric type elements represents the best compromise between accuracy and efficiency. The isoparametric elements are elements which use the polynomial interpolation, the variation order being the same for geometry and physical values. This type of elements are mainly used in the special literature and will be used in the software program implemented by the authors [2]-[5].

2. NUMERICAL MODELING SOFTWARE PROGRAM BASED ON BEM

2.1. Boundary element method principle

In order to describe the fundamental relations on which the formulation of a field problem through boundary element method is based, is considered the simple case of a closed 2D domain Ω bordered by the smooth boundary Γ , \mathbf{n} being the outward normal (figure 1).

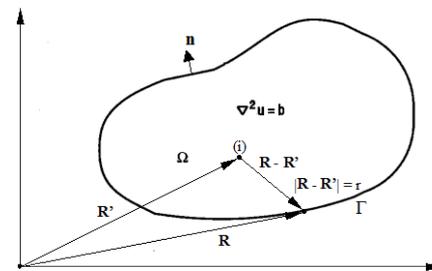


Fig. 1 Explanatory figure regarding the Boundary element method principle

Thus, to address a field numerical analysis problem through boundary element method, one must take into account as initial data, both the governing equation and the corresponding fundamental solution, and the value of the potential function or its derivative relative to the normal, in every calculation node of the boundary.

The method implies two stages, namely first it is necessary to determine the unknown magnitudes on the boundary, which in general case, have the expression:

$$c_i u_i + \int_{\Gamma} u \frac{\partial G}{\partial n} d\Gamma = \int_{\Gamma} G \frac{\partial u}{\partial n} d\Gamma - \int_{\Omega} G b d\Omega \quad (1)$$

where

$$c_i = \begin{cases} 1, & i \in \Omega \\ 1 - \frac{\Theta_2 - \Theta_1}{2\pi}, & i \in \Gamma \\ 0, & i \notin \Omega \end{cases} \quad (2)$$

if the boundary is smooth in the calculation point (i) – common practical case- then $c_i = 0.5$ in the boundary points. It must be also stated that if u represents the potential function satisfying a linear differential function, in the general case $\Delta u = b$, then G represents the fundamental solution corresponding to this governing differential equation of the studied problem which in the 2D case has the expression:

$$G(\mathbf{R} | \mathbf{R}') = \frac{1}{2\pi} \ln \frac{1}{r} \quad (3)$$

where \mathbf{R} is the position vector of the „calculation” point, \mathbf{R}' is the position vector of the „source” point and $r = |\mathbf{R} - \mathbf{R}'|$.

Thus, at the end of this stage in each calculation node the potential function value and its derivative relative to the normal value are both known.

The second stage follows, stage which implies the determination of the unknown magnitudes inside the domain with the general relationship:

$$u_i + \int_{\Gamma} u \frac{\partial G}{\partial n} d\Gamma = \int_{\Gamma} G \frac{\partial u}{\partial n} d\Gamma - \int_{\Omega} G b d\Omega \quad (4)$$

The practical solving of the integral equations as described before is made through the boundary discretization through boundary elements and the numerical evaluation of the integrals at each element's level. As it was mentioned above, the isoparametric elements are the most frequently used for the boundary discretization [6] – [10].

The main isoparametric boundary element types used in the special literature are the constant, linear and square type elements. Obviously, the possibility of using superior order elements exists, but as experience proves, the interpolation order increase is not

essentially reflected in the solution accuracy. On the other hand, even if the constant type elements are used, the calculation precision is very good, so the increase of the problem complexity and implicitly of the calculation time and necessary hardware resources is not justified in the vast majority of applications, thus in the developed program constant type isometrical elements were used. In the case of the constant elements the u values, $\partial u / \partial n$ are considered to be constant along an element and equal to the value corresponding to the middle of the element, where the calculation node is considered to be placed. In this case the points from the elements extremities are used only for the approximation of the problem geometry and the element is presented in the form of a straight segment. The integral (metric) terms calculation is made, in the vast majority of the applications from the special literature, using the Gauss quadrature method [11] – [16], thus in this case this method was also used.

2.2. The development of the numerical modeling software program *BEM2D*

The *BEM2D* program was implemented and developed in the Microsoft Visual Studio 2010, Visual C# program environment and has as a purpose the numerical modeling of electromagnetic devices. The general characteristics of the program are as follows:

➤ General data:

- number of maximum usable boundary elements: 800;
- boundary elements type: constant elements;
- maximum number of usable surfaces: 50;
- the possibility of using the predefined geometries: circle, rectangle, triangle type, and user defined segment by segment geometry;
- possibility of saving the application geometry in a data file and the possibility of further opening this application in order to modify it and it's numerical analysis;
- the possibility of generating a file with the calculation points in the domain (regular array of points);
- 'on-line' display of the geometrical characteristics (type, position), of the boundary conditions and of the number of boundary elements set for the application in work;
- total running time display;

➤ Used working domains:

- bounded domain (interior);
- 2D space infinitely extended;
- symmetry to earth (image type), 0x being the axis of symmetry (metallic surface of $V=0$ potential); in this case the calculations are made in the superior semi space ($y>0$);
- symmetry to the circular mantle, case used for the partial capacitances determination for the multi-wire cables;

➤ Calculation possibilities:

- capacitances calculation;

- potential values calculation in the points from the work domain and drawing spectral equipotential lines;
- electric charges calculation on the conductor surfaces;
- electrostatic energy calculation;
- electric field determination.

2.3. Presentation of the numerical modeling software program BEM2D

In order to facilitate the description of the operating mode, complexity and utility of the *BEM2D* program, an actual numerical example will be considered and the working mode will be followed, going through the necessary steps for its solving. Thus, solving an actual application using the software program *BEM2D* implies going through 5 main calculation stages which will be described as follows. *BEM2D* program has a friendly interface, similar to the commercial numerical modeling programs, fact which leads to a very easy application implementation.

As a numeric calculation example, two identical conductors with circular section and radius $R=0.01$ located near the earth surface were considered; the centers of the two conductors are in the points with the coordinates $(-0.02, 0.02)$ and $(0.02, 0.02)$ (figure 2). Knowing the potential values for the two conductors, namely $+100V$ and $-100V$, the potential values in the following points $P1(-0.01, 0.01)$, $P2(0, 0.01)$, $P3(0.01, 0.01)$ are wanted to be determined; also the electric charges for the two conductors and the partial capacitances for the two conductors are also determined.

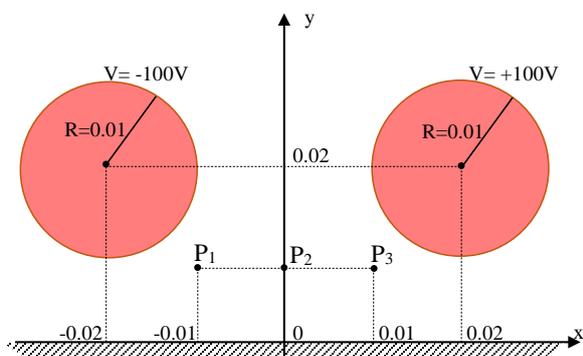


Fig. 2 Numerical example

The first stage implies entering the program, made in two steps, namely: first the numerical modeling type is chosen (2D or 3D) (figure 3), and then the project is defined, *BEM2D* program offering the possibility to create a new project and also to open an existent 2D project. The fact that at this moment only the 2D numerical modeling part for the electrical field analysis was developed must be said, the rest of the modules representing future research directions for the authors.

The second stage implies the problem description. For this the operating mode corresponding to the analyzed problem (figure 4.a), and corresponding to the

wanted application (figure 4.b) is chosen and the editing options for the application are activated. In this step, the wanted options for the application editing are set, fact which can be made: *Manually* (keyboard) or *Graphically* (surface and interior points drawing using the mouse and the drawing area).

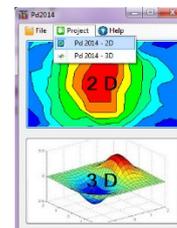
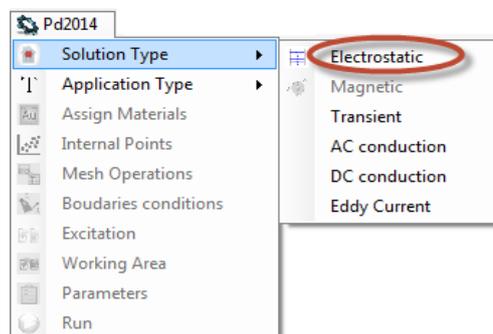
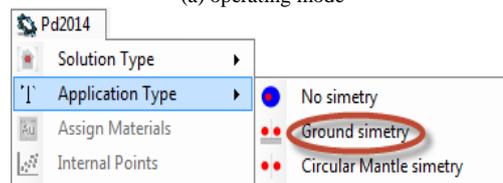


Fig. 3 Numerical modeling type 2D or 3D



(a) operating mode



(b) application type

Fig. 4 Choosing the operating mode and the application type

This step is followed by the geometrical model drawing, material assignment, boundaries and excitation assignment, work domain assignment and boundary discretization. Figure 5 presents a figure from the *BEM2D* program regarding the implemented model and the settings made until now.

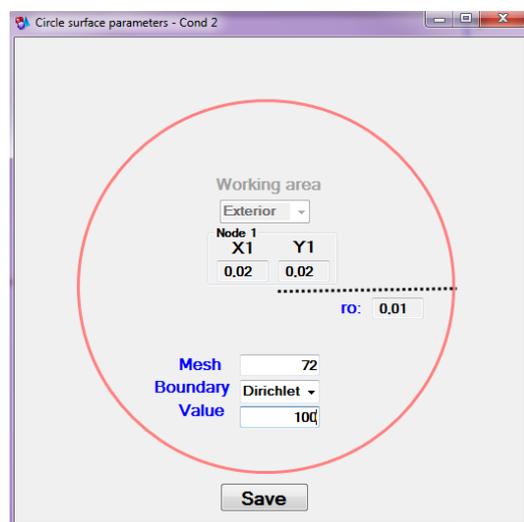


Fig. 5 Implementation description for the second conductor

In the third stage the calculation options activation takes place, fact which implies setting the internal calculation points, points in which the electric potential value calculation will be required for this case (figure 6).

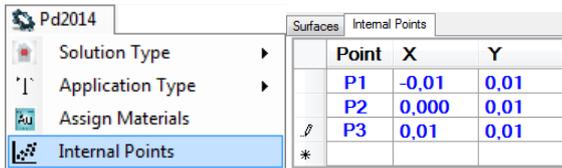


Fig. 6 Internal calculation points definition

Also at this stage the calculation executive parameters setting and the file corresponding to the internal calculation points from the domain generation setting takes place as it is presented in figure 7.

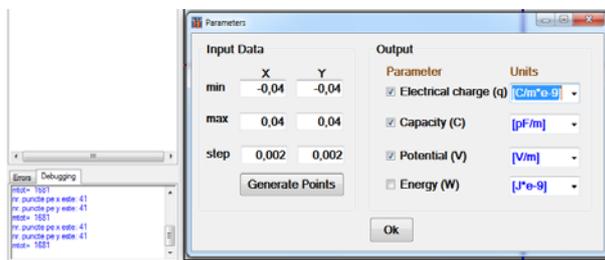


Fig. 7 Output parameters setting

At the end of this stage saving the data in a file whose name is given by the user is possible.

The problem's numerical analysis is made in the fourth stage. Thus, the end of the problem's parameter definition coincides with the validation of the button „Run” which allows the access to the run monitoring window for the actual numerical analysis module. The run monitoring window for the actual numerical analysis module is presented in figure 8. Here the information regarding the run progress and the running time are displayed.

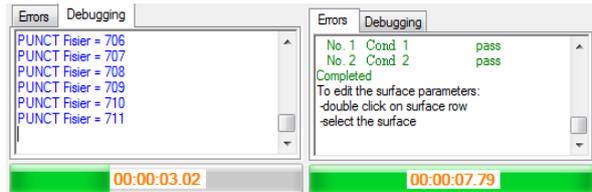


Fig. 8 Program BEM2D running window

In this stage the running of the numeric calculation module through BEM itself takes place. According to the statements above the corresponding boundary integral equation is given by relation (1) where $b=0$ and the coefficient $c_i=0.5$ are given for calculation point on the boundary and $c_i=1$ is given for a domain point.

Ending in good conditions the numerical analysis validates the buttons corresponding to the results view (figure 9) presented in stage 5.

The display window of the results is presented in figure 10. This window displays based on the options set in Stage 3 The activation of the calculation options the following magnitudes: electric potential in the specified internal calculation points; electric charges on the two conductors; partial capacitances, on the main diagonal in the result the capacitances to the ground C_{k0} and in the other terms of the matrix the capacitances between the conductors $C_{kj}=C_{jk}$ are shown.

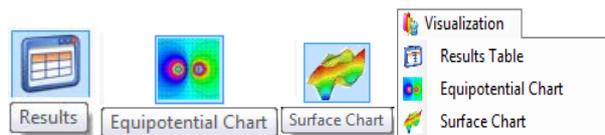


Fig. 9 Controls corresponding to the results display

In order to avoid the numerical calculation errors (and implicitly the graphical representation errors) the BEM2D program includes a special internal procedure for testing the affiliation of the calculation internal points red from the *intpct.dat* file to the work domain.

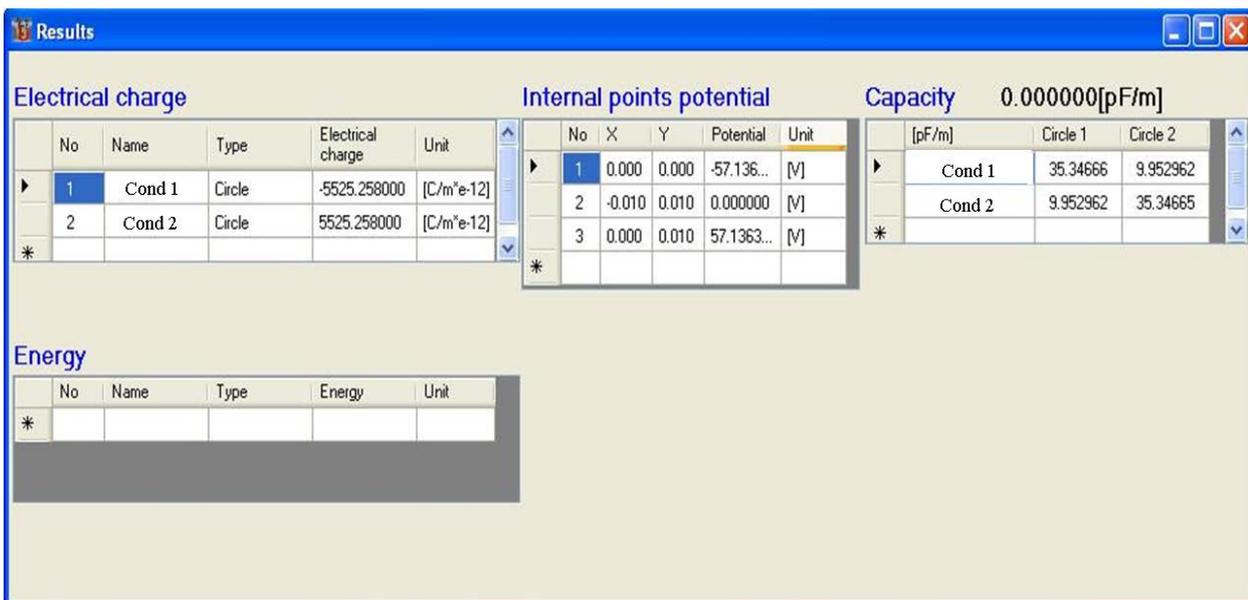


Fig. 10 Results window

Thus:

- a coding system through option semaphores „Interior – Exterior Domain” is selected for each device depending on their type, in accordance to the definition menus; thus it can be „identified” for each point from the internal points matrix their relative internal position relative to the implemented devices;
- if the Dirichlet boundary condition is constant on the entire boundary of the device, the device is considered to be conductor and, by case, the points from its interior or exterior are automatically recorded with the given boundary condition value; thus, for the considered example, the case of the interior circular conductor, for the internal points of the point matrix belonging within these conductors a value of +100V respectively -100V are assigned;
- if the problem is considered with symmetry to the earth, the internal points with negative horizontal axis ($y < 0$) are automatically assigned a value equal to zero.

3. TESTING THE NUMERICAL MODELING SOFTWARE PROGRAM *BEM2D*

In order to test the implemented numerical calculation software program based on BEM some 2D electric field applications were proposed and were solved using the software program *BEM2D* developed and implemented by the authors, and also using the commercial software program *Maxwell 2D Field Simulator SV* and through analytic calculation. In this paper a very simple example was considered, namely the case of an *ideal 2D plan capacitor*, determining the electric potential values in different points placed between its two armatures.

Details regarding the excitation assignment, the boundary conditions and model discretization are presented in figure 12.

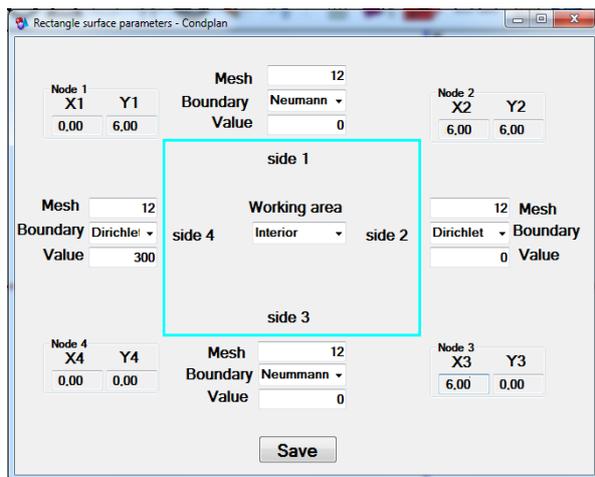


Fig. 12 Description of the model implemented in *BEM2D*

Also the program’s accuracy determination based on the number of used boundary elements is followed. Thus, for this application the electrical potential values

for the case of only 12 boundary elements (figure 13) and for the case on 72 boundary elements (figure 14) were determined.

No	X	Y	Potential	Unit
1	2.0	2.0	200.281	V
2	2.0	4.0	200.281	V
3	4.0	2.0	99.740	V
4	4.0	4.0	99.740	V
5	3.0	3.0	150.01	V

Fig. 13 Electrical potential values obtained with the *BEM2D* program for N=12 boundary elements

No	X	Y	Potential	Unit
1	2.0	2.0	200.024	V
2	2.0	4.0	200.024	V
3	4.0	2.0	99.994	V
4	4.0	4.0	99.994	V
5	3.0	3.0	150.005	V

Fig. 14 Electrical potential values obtained with the *BEM2D* program for N=12 boundary elements

In order to validate the *BEM2D* program, as it was mentioned above, this application was solved also with a commercial numerical modeling program *Maxwell 2D Field Simulator SV* using the same geometrical data, material properties, boundaries and excitations. The results obtained following the numerical modeling of this model are presented in figure 15.

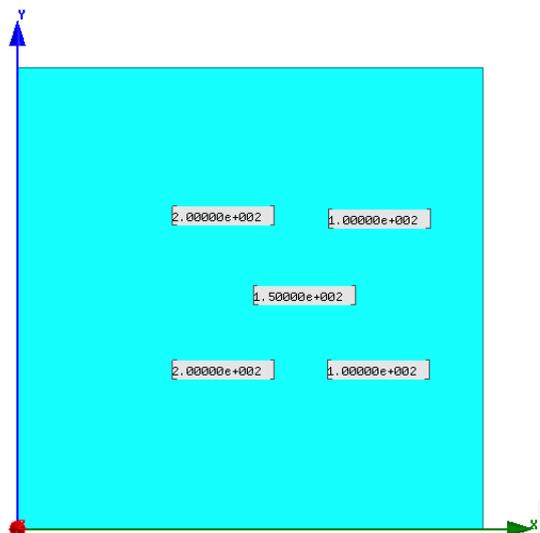


Fig. 15 Electrical potential values obtained with the program *Maxwell 2D Field Simulator SV*

At the end the electrical potential values in the 5 points of interest were determined also analytically. For a better visualization of the results in Table 2.1 and Table 2.2 the obtained results through the 3 methods are presented along with the calculation errors.

Table 1 – Comparison of the results for N=12 boundary elements

Calculation point	Analytical point [V]	Numerical potential <i>Maxwell 2D Field Simulator</i> [V]	Numerical potential <i>BEM2D</i> [V]	Relative error ϵ [%]
(2, 2)	200	200	200.2797	0.14
(2, 4)	200	200	200.2797	0.14
(4, 2)	100	100	99.7398	0.26
(4, 4)	100	100	99.7398	0.26
(3, 3)	150	150	150.0093	6E-3

Table 2 – Comparison of the results for N=72 boundary elements

Calculation point	Analytical point [V]	Numerical potential <i>Maxwell 2D Field Simulator</i> [V]	Numerical potential <i>BEM2D</i> [V]	Relative error ϵ [%]
(2, 2)	200	200	200.023	0.01
(2, 4)	200	200	200.023	0.01
(4, 2)	100	100	99.982	0.02
(4, 4)	100	100	99.7398	0.02
(3, 3)	150	150	150.003	2E-3

Thus, comparing the obtained results following the numerical modeling of this application using the numerical modeling program developed by the authors with the analytically obtained results and numerically obtained results using the commercial program mentioned above it can be concluded the fact that the numerical modeling program *BEM2D* implemented represents a numerical precision instrument.

4. CONCLUSIONS

In this paper the electromagnetic field numerical modeling software program *BEM2D* elaborated by the authors is presented. The paper begins with the short presentation of the Boundary element method on which the *BEM2D* is based and a presentation of its general characteristics. In order to highlight the complexity, utility and efficiency of this program the implementation stages of a new project with the *BEM2D* program for the case of an actual example are presented. At the end of the paper the testing and validation of the *BEM2D* program was shown. Solving of a problem both with the *BEM2D* program elaborated by the authors and with a commercial software program and analytically is proposed. Thus, following the obtained numerical results in the case of this application, it can be concluded that the implemented numerical modeling program *BEM2D* represents an extremely efficient tool for this category of problems, the convergence studies conducted fully justifying this claim.

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