ON THE dL ALGORITHM FOR CONTROLLING THE HYBRID SYSTEMS

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This paper discusses an algorithm for simulation, computation and control of the hybrid systems. The algorithm is known under the name *the differential dynamic logic* (dL). The hybrid systems are complex dynamical structures that couple the continuous motions with discrete motions of their constituents. Hybrid systems can be found in the biomedical industry, robotics, automotive, railway and aerial navigation. A system can flow or jump, can slip or collide with objects, or can be a cooperative surgeon-robot. A bouncing ball, for example, is an example of hybrid system that can exhibit continuous motions between each bounce and discrete motions between impacts to ground. This behavior describes a system with an infinite number of jumps in a finite interval of time. The dL incorporates the discrete and the continuous behavior of the system and can be used for computation and control of a hybrid system with uncertain boundary data modeled as a lognormal random field. This paper discusses the behavior of a cooperative surgeon-robot in which the coupling between the continuous motions of the surgeon and the discrete motions of the robot are incorporated in a single routine in which the computation, physical aspects and control are interacting.
Keywords: Hybrid systems; Differential dynamic logic control; Motion trajectories; Uncertain boundary data.

1. INTRODUCTION

Substantial progress was made in the last decade in computing and control of the hybrid systems characterized by interactions between the continuous and discrete motions of the components. The computation, physical aspects and control incorporate both the discrete and continuous sequences [1-5]. Details on this issue are found in [6-11].

In this paper, the dynamic differential logic algorithm (dL) is used to control the behavior of a cooperative surgeon-robot system having as main task the avoiding collisions with forbidden frontiers located in a known area [12-16]. A system with analog equipment is described by continuous equations, while a software system that processes the data, is described by discrete logic algorithms. An interface between these is realized by dL in order to control the motions and eliminate the unexpected situations [17, 18]. The control of the cooperative surgeon-robot system consists in stopping the surgical instrument (SI) to reach the prohibited frontiers. The motion control in the null-space of the contact force control is performed in a space coordinate defined as the normal direction to the contact point in a minimal space coordinate. The contact identification is analyzed by checking of the minimum distance between two bodies.

The algorithm dL performs the continuous dynamics described by differential equations which suffer perturbations and challenges due to uncertain boundary conditions. The model verification is an important step for performance and an efficient control [19].

The paper is organized as follows: Section 2 is devoted to description of the cooperative surgeon-robot system. The discrete and continuous behavior variables and the arithmetic operations handled by dL are presented in Section 3. The control of the motion trajectories of SI for different inputs and uncertain boundary conditions is presented in Section 4. The results are described in Section 5, while the conclusions are drawn in Section 6.

2. DESCRIPTION OF THE TASK

A hybrid system combines instantaneous discrete jump dynamics with continuous motions. Fig. 1 shows an example inspired from the train dynamics [1] with the acceleration a which changes instantaneously by discrete control interventions at some points in time, and of the continuous evolution of velocity v and the position z of the train. The differential equations of the train behavior are

$$\dot{z} = v, \quad \dot{v} = a \ .$$

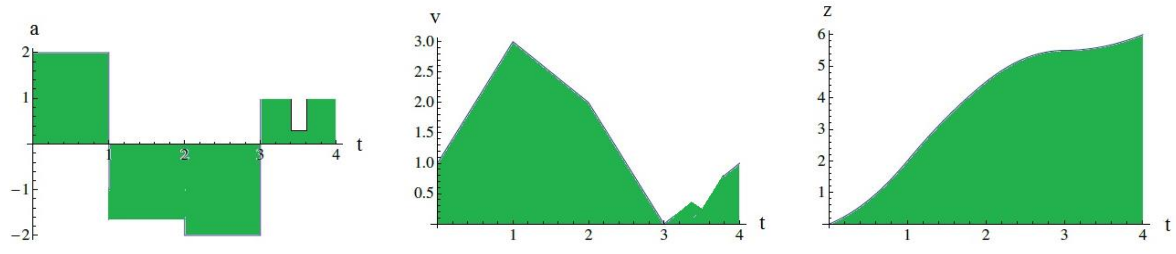

Figure 1 - An example of discrete evolution of acceleration, of continuous evolution of velocity and the position of a train over time.

In surgical interventions, the goal of the robot is to avoid SI to touch and intersect the prohibited frontiers in the working space Ω defined by coordinates (x, y, z), in order to help the surgeon to deal conflicting situations which could appear during surgical procedure on the SI trajectories to the final point T. The SI is viewed as the tip of a virtual joystick (red) (Fig.2). The surgeon hands freely the SI in Ω without robotic interventions, but, when SI located at the distance d to Γ, is in contact with the prohibited area $D < d$, the robot reduces the SI speed proportionally to D (Fig.1).

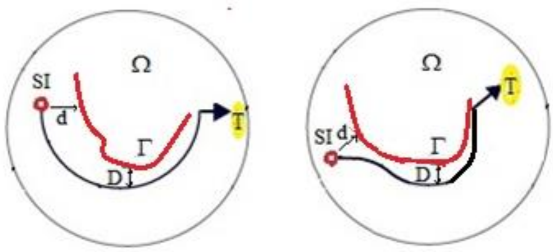

Figure 2 - Two ways of avoiding the prohibited frontiers in the surgery control.

The motion control in the null-space of the contact force control is performed in the space coordinate Π defined as the normal direction to the contact point in a minimal space coordinate [21]. The contact is analyzed by checking of the minimum distance between two bodies [22,23]

$$\min\left(\frac{1}{2}(r_1 - r_2)^T (r_1 - r_2)\right), \quad f_1(r_1) \le 0, \ f_2(r_2) \le 0 \ , \qquad (1)$$

where r_1 and r_2 are the position vectors of two points belonging to SI and tissue, respectively, and f_1 and f_2 are bounding surface constraints, respectively

$$\min(-d), \quad f_1(r_1) \le -\frac{d}{2} e_1, \quad f_2(r_2) \le -\frac{d}{2} e_2 \ , \qquad (2)$$

where d is the interference distance, and e_1 and e_2 are the unit vectors. We denote by J and n_c, the Jacobean and the contact normal vector

$$J_c = n_c J \ . \qquad (3)$$

The motion equations of the robot are [16]

$$A(q)\ddot{q} + b(q, \dot{q}) + g(q) + J_c^T(q) f_c = \Gamma \ , \qquad (4)$$

where $q=(q_1,q_2,q_3)$ is the joint vector, with q_1 the rotation about X -axis (pitch angle), q_2 the rotation about Y -axis (roll angle) and q_3 the rotation about the axis Z which is common with the joystick axis (yaw angle), respectively, $A(q)$ is the mass/inertia matrix, $b(q,\dot{q})$ is the Coriolis/centrifugal torque, and $g(q)$ is the gravity torque in the joint space, respectively.

The vector of joint torques Γ is a sum between the control torque for the contact force control and the torque in the null space of the contact force control

$$\Gamma = J_c^T f_c + N_c^T \Gamma_0 . \qquad (5)$$

The model of Hunt and Crossley (1975) is used for defining the contact force

$$f_c = k\delta^{\tilde{n}} + \tilde{b}\delta^p \dot{\delta}^q , \qquad (6)$$

where $\tilde{n}, p, q$ are constants, coefficient k depends on the material and the geometric properties of the bodies in contact, and $\tilde{b}$ is defined with respect to the coefficient of restitution $0 \le e \le 1$, $e = 1 - 2\tilde{b}\dot{\delta}_0 / 3k$ [24, 25]. For $\tilde{b}=0$, (6) reduces to the Hertz model.

The uncertain boundary conditions attached to (4) are described by random perturbations that can have a significant impact on the results. However, the boundary conditions depend on the tissues conditions and the geometry. The boundary conditions can be modeled as a lognormal random field given by

$$K(s_j, d, D_c, F_c, F_f, x, t) = \exp\big(Y(x,t)\big) , \qquad (7)$$

where x is the spatial coordinate, $Y(x,t)$ is a Gaussian random field with mean $<Y>=0$ and a correlation function $C_Y(x,y)$ associated to the variance σ_Y^2 and the correlation lengths of the random field Y

$$C_Y(x,y) = \sigma_Y^2 \exp\left(-\frac{|x_1-y_1|}{\eta_1} - \frac{|x_2-y_2|}{\eta_2} - \frac{|x_3-y_3|}{\eta_3} \right) . \qquad (8)$$

3. DESCRIPTION OF dL

The language dL is described by discrete and continuous state variables coupled by arithmetic operations: logical and $a \wedge b$; logical or $a \vee b$; negation $\neg a$; existential and universal quantifications in R, $\exists x P(x)$ and $\forall x P(y)$; a satisfying the condition $\psi\ [a]\psi$ satisfying $\psi\ <a>\psi$. The continuous motion is written as $\dot{x}_1 = \varphi_1, \dot{x}_2 = \varphi_2, ... \dot{x}_n = \varphi_n \ \&\ \psi$ completed with the assumption ($?\psi$), the assignment ($x_i := \varphi_i$), the non-deterministic assignment of any value ($x_i := *$), the sequentially running a and b ($a;b$), the non-deterministic choice ($a \cup b$) and the non-deterministic loop ($a*$) [1-5].

For example, an arbitrary input of a non-deterministic value to f of dL is [26]

$$\begin{aligned}
&\text{ctrl} = (f := *;\\
&(\dot{r} = gf \ \&\ f >= 0) \cup\\
&(\dot{r} = gf \ \&\ (f <= 0) \wedge (r >= D)) \cup\\
&(\dot{r} = g(r/D) f \ \&\ (f <= 0) \wedge (r <= D)))*
\end{aligned} \qquad (13)$$

The hybrid programs can combine discrete and continuous transitions to different structured control programs using the operators of Kleene algebras. For example, the simplified train control is written as

$q := accel; \quad /{*}initial\ mode\ is\ node\ accel{*}/$

$(\ (?q = accel; z' = v, v' = a)$

$\cup(?q = accel \wedge z \geq s; a := -b; q := brake; ?v \geq 0)$

$\cup(?q = brake; z' = v, v' = a \,\&\, v \geq 0)$

$\cup(q = brake \wedge v \leq 1; a := a + 5; q := accel))*$

The SI is starting from a location $(r >= 0)$, continues his task in Ω, and its safety motion is written as $(r >= 0) \rightarrow [\mathrm{ctrl}(r >= 0)$. (14)

The sequence (14) is the key of KeYmaera, a useful instrument that can check the safety property of the algorithm [2,5].

The constraints are modelled in linear or nonlinear inequalities over Boolean-valued variables

$formula ::= \{clause \wedge\} * clause$

$clause ::= linear\_consta\,\mathrm{int}\,s \,|\, boolean\_var \rightarrow linear\_constraint \cup$

$clause ::= nonlinear\_consta\,\mathrm{int}\,s \,|\, boolean\_var \rightarrow nonlinear\_constraint$

4. CONTROL OF CROSSING THE PROHIBITED BOUNDARIES

The surgical robot built at the Johns Hopkins University Center for Integrated Surgical Systems and Technology Group [17, 18, 27, 28] is composed of three components: Stealth Station navigation unit that follow the position and orientation of the optical markers on the rigid body, the 3DSlicer unit for viewing and analyzing imaging data, and a 6-degree Neuromate robotic arm equipped with a Food & Drug Administration (FDA).

We imagine to have such a robot in order to describe how to use dL for simulation, computation and control of such robot. When the surgeon applies a force f to SI, we have [6-8]

$$\frac{\mathrm{d}r}{\mathrm{d}t} = G(f), \tag{15}$$

where $r > 0$ describes the SI position in Ω, and G a constant multiple of f. The analyze of SI trajectories and the crossing of a prohibited boundary Γ is done by applying the Greenwood and Novikov results [29-31]. In the 1D case, Ω is bounded by a constant boundary $-g$, $g \geq 0$, and the motion is described by

$$s_n = s_0 + \sum_{p=1}^{n} x_p , \tag{16}$$

with $x_p, p \geq 1$ the state variables. We suppose that SI reaches the prohibited border at the time t_g

$$t_g := \min\{n \geq 1 : s_n < -g\}. \tag{17}$$

By checking the minimum distance between SI and Γ we identify the contact

$$\min\left(\frac{1}{2}(r_1 - r_2)^T (r_1 - r_2)\right), \tag{18}$$

where r_1 and r_2 are the position of SI and the border, respectively. The class $M \in R^2$ of given motions of SI is generated by a genetic algorithm

$$M := \{0 < \mu < 1; |\nu| < 1\} \cup \{0 < \mu < 2; |\nu| \leq 1\} \cup \{\mu = 1, \nu = 0\} \cup \{\mu = 2, \nu = 0\}, \tag{19}$$

The speed $\dot{r}_1$ is given by [26]

$$\dot{r}_1 = \dot{r} - \left(1 - \frac{d}{D}\right)(\dot{r} \cdot n_1) n_1 , \tag{20}$$

where d is the distance from SI to Γ with the normal n_1.

The control of crossing the prohibited frontiers is described by

$$
\begin{aligned}
& ctr_2 = (\\
& f_{1p} := *; f_{2p} := *; \\
& f_{np} := (f_{1p}n_1 + f_{2p}n_2); \\
& f_n := (f_1 n_1 + f_2 n_2); \\
& d_0 := ((q_1 - px)n_1 + (q_2 - py)n_2); \\
& dist := (d_0 + k(f_n e + (f_{np}e^2)/2)); \\
& disc := ((kf_n)^2 - 2kf_{np}d_0); \\
& (?\, f_{np} \le 0 \wedge dist \ge 0; \\
& g := 0) \\
& \cup(?\, f_{np} \le 0 \wedge dist \ge 0; \\
& g := (f_n + (d_0 + (kf_{np}e^2)/2/(ke))) \\
& \cup(?\, f_{np} \ge 0 \wedge f_n \le 0 \wedge dist \le 0; \\
& g := 0) \\
& \cup(?\, f_{np} \ge 0 \wedge f_n \le 0 \wedge dist \ge 0 \wedge f_n + f_{np}e \ge 0; \\
& g := (f_n - Sqrt((2d_0 f_{np})/k))) \\
& \cup(?\, f_{np} \ge 0 \wedge f_n \le 0 \wedge disc \ge 0 \wedge f_n + f_{np}e \le 0 \wedge dist \le 0; \\
& g := (f_n - Sqrt((2d_0 f_{np})/k))) \\
& \cup(?\, f_{np} \ge 0 \wedge f_n \le 0 \wedge disc \ge 0 \wedge f_n + f_{np}e \le 0 \wedge dist \le 0; \\
& g := 0 \\
& \cup(?\, f_{np} \ge 0 \wedge f_n \ge 0; \\
& g := 0); \\
& t := 0; \\
& (q_1' = k(f_1 - gn_1), q_2' = k(f_2 - gn_2), f_1' = f_{1p}, f_2' = f_{2p}, t' = 1 \,\&\, t \le e)
\end{aligned}
\tag{21}
$$

Here, the distance from Γ is d, the unit normal to Γ is n_1 and the velocity p' is given by the control law

$$p_1' = p' - \left(1 - \frac{d}{D}\right)(p' \cdot n_1)n_1 \,, \tag{22}$$

where p' is the speed of SI and dot product of two vectors is denoted by " $\cdot$ ".

Most of the time, the surgeon manages freely the SI (green circle) without any robotic intervention (blue lines). Various situations with the surgeon's given trajectories in space are presented in Fig.3. The critical border Γ is the ends of Ω marked with red colour.

When SI approaches Γ, the normal speed component to the border is reduced by the robot and slowly cancelled. It is the case of the regions noted by A and B. It is possible that some SI trajectories not to be defined ab initio, and to be instantly changed depending on local working conditions. Or, the inputs can be unclear and the measurement imperfect. Such situations are handled by intervention of the robot and the surgeon accepting.

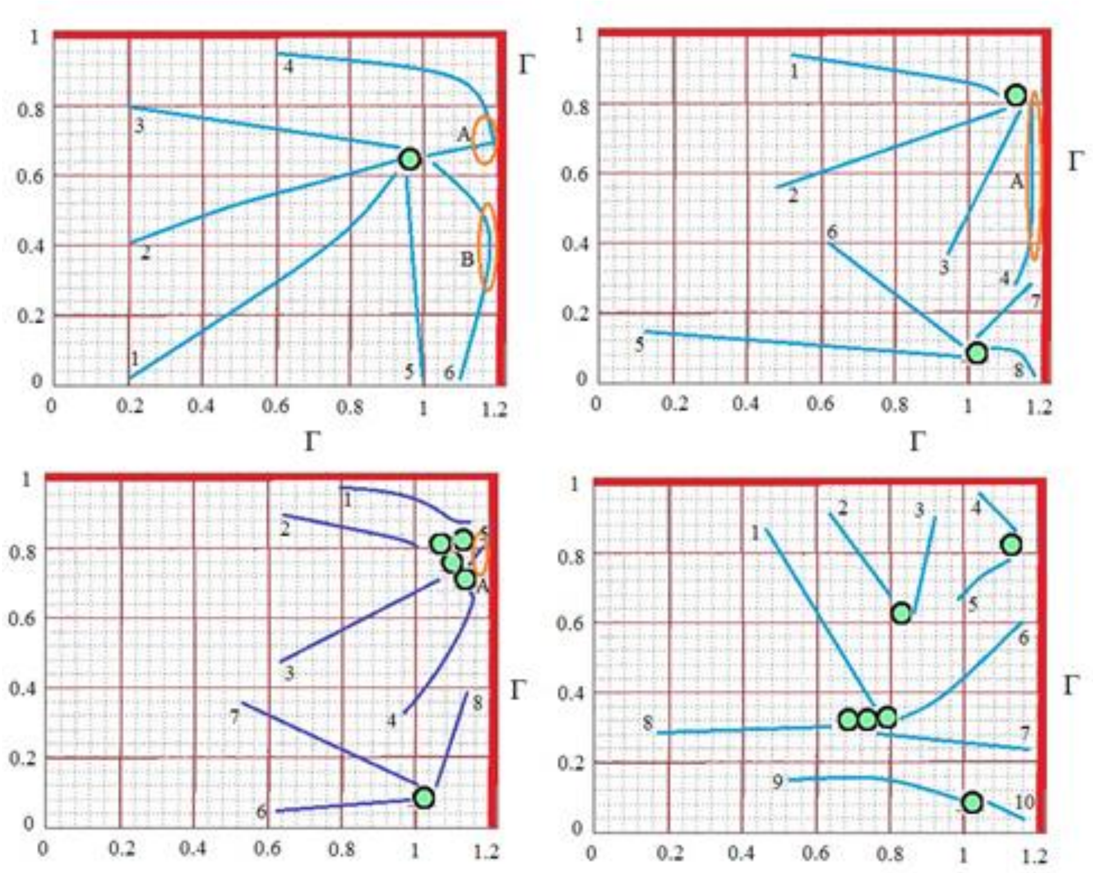

Figure 3 - Different SI trajectories in Ω.

The implementation of hard or soft stops takes into account on the magnitude of the force and the distance to Γ, and verifies the condition do not slide across Γ. The proof of the control algorithm is a very important step in this simulation, and this must verify preconditions and also postconditions relative to initial set of different branches of the loop and its invariant. We add that in our case there are seven branches, one for each of the different input cases we considered. The control algorithm can be proved with respect to its safety using KeYmaera [12, 32].

5. CONCLUSIONS

The differential dynamic logic dL is used in this paper for computation and control of a hybrid system with uncertain boundary data. The hybrid system is a cooperative surgeon-robot in which the coupling between the continuous motions of the surgeon and the discrete motions of the robot are incorporated in a single routine in which the computation, physical aspects and control are interacting. dL incorporates the discreet and the continuous behavior of the system, and the proposed algorithm is able to analyze possible uncertain boundary data attached to the model. The uncertain boundary data are expressed as a lognormal random field.

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